Shodhbodhalaya: An International Peer reviewed Multidisciplinary Journal

Volume 4 | Issue 3 | ISSN: 2584-1807 | (https://shodhbodh.com/)

An Optimal Inventory Model with Stock-Dependent Demand, Retroactive Holding Costs, and Partial Shortages

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Abstract

This study proposes an advanced mathematical model of the inventory which loosens the conventional assumptions, combining inventory-level dependent demand, variable holding cost and partial shortage. In most classical inventory models, demand is assumed to be constant and the storage costs are constant without considering the dynamics of perishable goods and the demand rate responding to the visibility of stock. This is the reason the proposed model represents demand as a linear function of the inventory available, which represents the elasticity of demand affected by the product availability. In addition, the model incorporates a holding cost that is a time dependent step function with retroactive applicability. This will allow the increasing costs of maintenance, depreciation and spoilage to be properly reflected during the full duration of storage. The system also allows shortages, which are entirely backlogged and fulfilled when needed, and this is an effective way for businesses to adjust their fulfillment times while preventing the risk of unnecessarily having to pay a higher holding cost. The computational model enables the determination of the Optimal Order Quantity and the Optimal Cycle time by minimizing Total Inventory Cost (TIC) which includes ordering cost, holding cost and shortage cost. The analytical results show that the higher the holding costs, the lower the optimal order quantity and cycle time will be to keep the cost efficient. Overall, this framework offers a comprehensive decision-making process for supply chain managers and retailers, especially for managing inventory of deteriorating products, reducing financial losses due to extended storage periods, and managing stockouts effectively in a dynamic market.

Keywords: Inventory Management, Stock-Dependent Demand, Variable Holding Cost, Retroactive Cost Increase, Partial Shortages, Backlogging, Total Inventory Cost (TIC), Deteriorating Items

1. INTRODUCTION

Past inventory models were built on the premise that demand is a static and constant variable and does not influence the amount of inventory a business has. This assumption is useful for mathematical modeling and helps to use inventory management methods in various situations [1]. But this is a simplistic model, and in a real-world situation, it doesn't necessarily reflect the true dynamics of a marketplace. In reality, the demand may not be the same as stated above, as it may vary with the availability of the product in the market, and a variety of other factors [2]. One of the important learning points from the practical application is that the demand for the product can also rise when there is more stock of the product on the shelves. This is often known as the phenomenon of demand elasticity, which means that the demand for a product is related, not only to the external variables such as the price of goods and services or the market conditions, but also to the internal variables of the business itself, such as their inventory policies [3-4]. For instance, when a retailer stocks more of a particular product, it may trigger higher demand due to increased visibility and accessibility of the product to consumers.

This is an important elasticity of demand in industries that produce products with a short useful life, which tend to become spoilt with age, or are seasonal [5-6]. A fruit market, for instance, might experience a surge in demand for fresh fruit when there is plenty available, but this demand could drop significantly as the availability of fresh fruit decreases or as the products start to spoil. Likewise, the fashion industry sees some clothing brands, for example, being more popular when they have a lot of merchandise in stock, but then less popular as the season progresses or stock levels run low. The traditional inventory models' assumption of constant

demand is a crucial one that fails to capture this relationship between stock and consumer behaviour. Demand is regarded as exogenous and outside of the control of the business, such as restocking or pricing. A steady, predictable demand curve that a certain amount of output is sold in a unit time period can be assumed in these models [7]. This model does not account for the fact that demand may increase or decrease depending on the quantity of stock generally available - especially when there is a lot of inventory changing hands. The amount available on the shelves can influence the price change, promotions and even consumer choice. Some of these factors are not considered in the classical models [8].

Since this is the case, new inventory models have been created that incorporate the inventory-level dependent demand. The models cover the notion that if more product is available for sale, more people will want to buy it, and vice versa. Not only does this match more closely what is observed in reality but it can also enable businesses to have a better prediction of sales and inventory requirements [28]. Another factor that is important besides the inventory dependent demand is holding cost. The concept of holding cost is assumed to be a constant value per unit of inventory regardless of the inventory holding period, for the whole inventory cycle, in the traditional models [9]. This is not true for products in the real world, however, especially for products that spoil such as food or chemicals. Holding costs can also rise with longer holding times, resulting in further costs for the company.

Often the cost of holding the product is not fixed, but depends on the time the product is in storage, or requires additional resources to keep the product in storage (e.g., refrigeration for perishables). Also, inventory value decreases with time, as a product nears its shelf life or expiration date the actual cost of inventory will increase [10]. This can create a situation where holding costs are rising with the passage of time, and it becomes more costly to carry an inventory. Shortages are a real-world problem as well as raising holding costs. Traditional inventory models assume that there are no shortages in the business, thus the business always

has enough inventory on-hand to satisfy demand. But in real life, this is not often the case as backordering and stockout are common in various industries the world over. In a situation of scarce supply and high demand, businesses may find themselves either having to store inventory in case of future orders or end up with backorders, in which case customers must wait for the product to be available again. This can lead to costs that can be incurred as well, such as losing customers to other companies and having poor customer satisfaction scores.

In this model, we extend the set of assumptions of previous models to incorporate inventory-level dependent demand, variable holding costs and shortages. The purpose is to simulate the actual dynamics of the inventory [29-31]. The holding cost for an item is assumed to be an increasing step function of the number of days the item is in the store. This is especially useful when the product deteriorates with time and the cost of storing a product increases as time passes [11]. For instance, the cost of holding inventory of perishables (such as fruit and vegetable products, dairy products, etc.) increases with the passage of time as the product goes bad. Cost may also go up due to the fact the business must cover the costs of product that is becoming spoiled, refrigerate the product, or pay more for labor to process the products at their end-of-life. The model takes these real-life constraints into account, aiming to help businesses optimise their stock, reduce expenses and ensure that they can deliver product to customer demand cost effectively.

This model is also advanced by including the shortage which is not included in traditional models, as well as other factors such as demand elasticity and holding costs. Backordering can be used to meet demand when stock is depleted and can be used to achieve a balance between the costs of carrying excessive inventories and meeting demand. The use of shortage and inclusion of backorder in the inventory management system helps businesses make better-informed decisions regarding the level of inventory to keep in stock and the most effective way to manage their supply chain.

This model aims to develop a more dynamic and realistic way of managing inventory – a way more similar to what businesses would do in the real world. It offers a structure for companies to efficiently manage their ordering quantities, stock levels and cycle times, whilst taking into account the multifaceted relationship between demand, stock holding costs and the risk of stock-outs.

Many inventory models have been inaccurate and inapplicable because of the traditional assumption that demand is constant and independent of inventory decisions. This paper proposes a model that overcomes these drawbacks by integrating inventory dependent demand, variable holding costs and shortage conditions. The inventory-level dependent demand is a faithful representation of the actual buying behavior of customers, where demand for products can be influenced by their inventory levels. Likewise, this model accounts for variable holding costs and shortages, which makes it a more realistic model for businesses to make optimal inventory management decisions. These improvements create an effective model for more accurately forecasting demand, reducing holding costs and managing inventory in line with real-world dynamics.

2. PROBLEM DEFINITION AND SCOPE

In this paper, the goal is to formulate an optimal inventory policy for a system where the demand rate is dependent on the inventory level, the holding cost is time- and level-dependent, and stock-out can be tolerated. The traditional inventory models are often based on the assumption of constant demand rate and holding cost for inventory during the entire inventory cycle, whose simplicity is not sufficient to capture the complexity of real world inventory system and has simplified the decision making process. The demand for inventory is often sensitive to the level of inventory, and the inventory holding cost may vary with the length of time that goods are

stored. Also, businesses may have a situation where shortages are unavoidable, and the cost of shortages must be controlled as a component of an inventory strategy [12].

This model extends these practical complexities by adding inventory level dependent demand, variable holding costs, and shortages into a holistic model of inventory management.

Inventory-Level Dependent Demand

The demand rate is usually assumed to be fixed in traditional inventory models and is not dependent on the inventory level. In some markets, this is true, but many markets do not follow this rule, and as the number of items available to buy grows, so can demand. This is known as inventory level dependent demand [13].

Take a retailer who carries a certain item, but only has a small amount. The retailer might have slow sales if the product's demand is fairly inelastic. The more the retailer has in stock, though, the more available the product is to the customer, and the more demand is generated [27]. The effect is greatest in retail stores where people can see the product in front of them; when stock is high, people will feel more inclined to buy it.

In this model, the demand rate is assumed to depend on the level of inventory with an increase in inventory having an effect on an increase in the demand rate [14]. This method takes into consideration the actual connection between stock and customer habits which can be that when stock rises, demand rises and when stock decreases, demand decreases. Thus, the inventory system is no longer under the control of a static demand curve, but instead under the control of a dynamic demand/inventory relationship.

Variable Holding Cost

A cost that is another factor in this model is the cost that a business incurs to keep inventory. In the traditional models the holding cost is assumed to be constant, this makes the calculations easier but does not take into account the complications of real conditions for the storage. The cost of carrying inventory doesn't always remain the same in industries and sometimes goes up because of the longer the product stays in the inventory, the more costly it is. This phenomenon is especially relevant in industries where perishable products or products with limited shelf life are used such as food products, pharmaceuticals, or chemicals [15].

We assume that the holding cost per unit is time dependent in our model, i.e., holding costs increase with the longer an item remains in storage. The holding cost could be a storage fee, insurance premiums, maintenance costs, and possibly depreciation, which may all rise as the item sits on the shelf or in the warehouse. For example, the storage costs of perishable items, such as fruits and vegetables, increase as the product nears expiration or refrigeration- or special-handling-required products cost more to store the longer they sit in inventory.

To further sharpen this model, we will make the assumption that the holding cost in the unit is a step function that increases with the length of the time that a product is stored. This indicates that the holding cost is raised after a number of discrete time periods (some pre-defined). The increase is not linear, but rather occurs in steps, which indicate changes in the storage conditions and/or needs as items get older [16]. This way, the real-life situation in which holding costs rise after a certain point; for example, when inventory moves to a higher cost category because of its age or unique storage requirements can be more accurately simulated by the model.

The unit holding cost could, for instance, be based on a constant allotment for the first 10 days, though the product has a 30-day life. Then, the price goes up in increments as the product comes closer to expiration. This step function approach allows to include the different costs of the

inventory that changes based on the different phases of the inventory, which will help in making better decisions, as it will capture the different financial implications of inventory over time.

Retroactive Increase of Holding Costs

In many inventory systems (particularly those involving perishable products and deteriorating products), holding costs can rise as the time goes on, sometimes increasing even retroactively. If the unit holding cost increases (due to extended storage), this new holding cost is now applied to the entire length of time the item has been in storage, not from the time the step increase occurs.

For instance, a product will have a holding cost of 30 per cent in store and then increase by 30 per cent for the subsequent 30 days period, as well as being retroactively applied to the previous 30 days. This is true for many business scenarios where the inventory costs are dependent on the future conditions and the duration of the product in the inventory [17].

A retroactive raise in holding costs may play a significant role for products that are affected by the time they are in inventory. A company may have pharmaceutical products in storage that need refrigeration, for instance, and if these products are stored for a long time, the costs of storing the pharmaceuticals (e.g., energy, special handling, etc.) may go up not only for future storage but also for the previous storage time. Since it is a retroactive adjustment, it makes sure that businesses cover all potential costs of inventory such as extra costs of keeping inventory for longer than anticipated [18].

Shortages

The other important component of such a model is the possibility of shortages. The usual approach to avoiding shortages in the traditional inventory models is to make sure that the demand is always satisfied with adequate inventory. But, in practice, it's hard to achieve 100%

availability of stock. Shortages happen all the time, whether it's due to unexpected surge in demand, disruption of supply chain or any logistical problem.

The addition of the possibility of shortages adds a new twist to the inventory model. When stocking is low, it may result in backordering, which means that if consumers can't get the product right away, they will have to wait for it to be replenished [19]. Shortages indicate that the business could have to make strategic choices between the cost of inventory and stock out costs. Having more stock on hand can help minimize the risk of running out of stock [26] but can add to holding costs. However, with less stock in the business, it can cause stockouts that may lead to lost sales, dissatisfaction and harm to the brand image of the business [21].

Modeling assumptions are that there are no holding costs during shortages. This assumption is that businesses have no cost associated with holding any inventory that is not in stock; and that if there is a shortage of some kind, there is no associated holding cost. The model still will account for lost sales cost, backordering cost and cost of customer dissatisfaction, though, which could be represented as part of the total cost of inventory management [20. 22].

Scope of the Model

The main objective of this model is to identify the optimal inventory policy that can be adopted to minimise the total cost of inventory management, which takes into account the inventory level dependent demand, variable holding cost and stock-out cost. The model will be a guideline for the businesses that optimize their order quantity, inventory level and ordering frequency and reduce the cost of holding the inventory or running out of stock while at the same time balancing the other costs associated with the business [23].

The model includes these factors—inventory-dependent demand, variable holding costs, retroactive holding cost increases, and allowance for shortages—in order to provide a more

comprehensive and realistic inventory management model for business. It will find the right balance between having enough stock to satisfy customers' needs and keeping inventory levels low to reduce the costs of holding excess stock, and will ultimately improve all inventory management activities.

3. ASSUMPTION AND LIMITATIONS

- Replenishments rate is infinite.
- Replenishment size is finite.
- Lead time zero.

Each time the order quantity is placed in Q quantity units, the order level will be set at the predetermined level of S and once the backorder quantity Q-S is satisfied, the order quantity will be placed again.

Inventory level reaches to zero at time t and after this time shortages takes place

The demand rate R is linearly increasing function of the inventory level I(t).

The holding cost is varying as an increasing step function of the quantity in storage.

A shortage may not have been cleared up, if any, and they are satisfied if new lot arrives.

One product is taken into consideration.

The demand rate R is linear function of the inventory level I (t) which can be expressed as: $R(I(t)) = R + \beta I(t)$; R (Const. demand) > 0, $0 < \beta < 1$, $(t) \geq 0$.

$-R\, I(t)$ indicates that demand during this time period is decreasing over time.

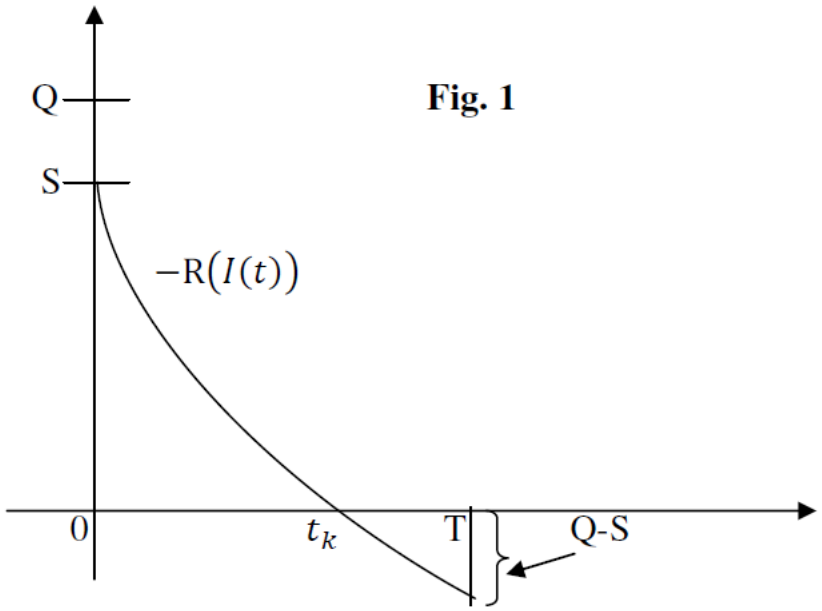

Fig. 1

4. INVENTORY MODEL

Our main objective is to minimize the TIC per unit time, which includes three components: The ordering cost, the holding cost and the shortage cost. We know at the start of the cycle ordering cost. We need to determine shortage cost and holding cost which we do as follows:

• Since one order is made per cycle, the ordering cost per unit time is given as A/T.

As we have assume that the shortages occurs at time t_k we can write I (t_k) = 0.

Holding cost per cycle can be obtained as follows:

$$IHC\int_0^{t_k} h(t)\,I(t)\,dt \qquad 1$$

Q quantity ordered and realized in stock and after fulfilling the back orders Q-S the order level reaches to the order level S.

At the end of the cycle the inventory level reaches to Q-S.

The instantaneous states of I(t) in the interval (0, T) is given by

$$\frac{dI(t)}{dt}=\begin{cases}-(R+\beta I(t)); & 0\le t < t_k \\ -R & ; t_k \le t < T\end{cases} \qquad 2$$

Here – 9 shows that there is a constant demand rate (Shortages) because there were no items in stock. This negative demand will be backlogged and satidfied when new stock arrived [24].

From 2 we can write

$$\frac{dI(t)}{(R+\beta I(t))}=-dt$$

Let us integrate both sides within (o, t) as follows:

$$\int_0^t \frac{dI(t)}{(R+\beta I(t))}=\int_0^t -dt$$

$$\frac{1}{\beta}\left[In(R+\beta I(t))-In(R+\beta I(0))\right]=-t$$

$$\therefore In\left(\frac{R+\beta I(t)}{R+\beta S}\right)=-\beta t$$

$$\therefore R+\beta I(t)+(R+\beta S)e^{-\beta t}$$

$$I(t)=\frac{1}{\beta}\left[(R+\beta S)e^{-\beta t}-R\right];0\le t\le t_k \qquad 3$$

Now from 2

$$\int_{t_k}^{t} \frac{dI(t)}{dt} = \int_{t_k}^{t} -R$$

On solving this we get

$$I(t) = -R(t - t_k); t_k \le t \le T \qquad 4$$

from 3 and 4 we can write I (t) as follows:

$$I(t) = \begin{cases} \frac{1}{\beta}\left[(R + \beta S)e^{-\beta t} - R\right]; 0 \le t \le t_k \\ -R(t - t_k) \qquad\qquad ; t_k \le t \le T \end{cases} \qquad 5$$

Now from 5

$I(T) = -R\,(T - t_k) = Q - S$

$$\therefore T = t_k - \frac{Q - S}{R} \qquad 6$$

$$\therefore Q = S - R(R - t_k) \qquad 7$$

Since at $t = t_k$; $I(t) = 0$ using 5 we can write

$$\left(\frac{R}{\beta} + S\right)e^{-\beta t_k} - \frac{R}{\beta} = -R(0) = 0$$

$$\therefore e^{-\beta t_k} = \left(\frac{R}{\beta} + S\right)\frac{\beta}{R}$$

$$\therefore t_k = \frac{1}{\beta} In\left[\left(\frac{R}{\beta}+s\right)\frac{\beta}{R}\right] \quad 8$$

Cost of Shortages

$$SC = -c\int_{t_k}^{T} I(d)\,dt$$

$$= -c\int_{t_k}^{T} R(t-t_k)\,dt$$

$$SC = -cR\left[\frac{(T-t_k)^2}{2}\right] \quad 9$$

5 RETROACTIVE HOLDING COST

In this case holding cost of last storage period applies retroactively to all previous periods, thus $h_{k\ is}$ the holding cost applied to all the previous periods because after the period t_{k-1} to t_k shortages takes place [25].

Holding Cost= $H = h_k\int_0^{t_k} I(t)\,dt$

$$= h_k\int^{t_k}\left[\left(\frac{R}{\beta}+S\right)e^{-\beta t} - \frac{R}{\beta}\right]dt$$

$$\therefore H = h_k\left\{\frac{1}{\beta}\left[\frac{R}{\beta}+s\right](1-e^{-\beta t}) - \frac{R}{\beta}t_k\right\} \quad 10$$

TIC = Ordering Cost + Shortage Cost+ Holding Cost

Now put the values of SC and HC from 9 and 10 we get

$$TIC = \frac{A}{T} + \frac{-cR}{T}\left[\frac{(T-T_k)^2}{2}\right] + h_k\left[\frac{1}{\beta}\left[\frac{R}{\beta}+S\right](1-e^{-\beta t}) - \frac{R}{\beta}t_k\right] \quad 11$$

Substitute the value of T from 6

$$TIC = \frac{R}{Rt_k - Q + S}\left[A - \frac{c(Q-S)^2}{2R} + h_k M\right] \quad 12$$

Where, $M = \frac{1}{\beta}\left(s+\frac{R}{\beta}\right)(1-e^{-\beta t})\frac{R}{\beta}t_k$

Now differentiating with respect to Q and equating to zero we get the following

$$\frac{\partial(TIC)}{\partial Q} = \frac{-c((Q-S)Rt_k - Q + S) + R\left(A - \frac{c(Q-S)^2}{2R} + h_k M\right)}{(Rt_k - Q + S)^2} \quad 3$$

Now since $\frac{\partial(TIC)}{\partial Q}$ = 0; we get

$$c(Q-S)(Rt_k - Q + S) + R\left(A - \frac{c(Q-S)^2}{2R} + h_k M\right) = 0$$

$$cR(Q-S)t_k - \frac{c(Q-S)^2}{2} = RA + RMh_k$$

$$\frac{c}{2}\left[-(Q-S)^2 + 2R(Q-S)t_k + R^2 t_k^2 - R^2 t_k^2 = RA + RMh_k\right]$$

$$(Q-S+Rt_k)^2 = -\frac{2}{c}(RA+RMh_k)+R^2t_k{}^2$$

$$Q = S-RT_k+\sqrt{R^2t_k{}^2-\frac{2}{c}(RA+RMh_k)} \qquad 4$$

6 ACTUAL OBSERVATION

$$\frac{\partial^2(TIC)}{\partial Q^2} = \frac{-c(Rt_k-1)+R\left(A-\frac{c(Q-S)}{R}\right)}{(Rt_k-Q+S)^2} + \frac{2\left\{-c((Q-S)Rt_k-Q+S)+A-\frac{c(Q-S)^2}{2R}+h_kM)\right\}}{(Rt_k-Q+S)}$$

Now on putting the value of Q from the 4 in to $\frac{\partial^2(TIC)}{\partial Q^2}$ we observe that the value of $\frac{\partial^2(TIC)}{\partial Q^2} > 0$ it means that the value of Q obtained in 4 gives us minimum value of the TIC which is required to us [25].

The following table shows that the optimal order quantity Q and the cycle time T decrease when the holding cost increases.

Let us take the parameters as follows:

R = 470 units, A = 250 per order, Elasticity of demand = β = 0.2, Holding cost, Cycle time and time at which inventory becomes zero are shown in the following table.

Holding Cost (h) per unit per time unit (In Rupees)	Cycle time (T)	Time at which inventory becomes zero (t_k) (In years)	Optimal order quantity (Q)	TIC
1	0.323	0.4	374	15443
2	0.642	0.8	371	1601.25
3	0.876	1.2	364	1648.53
4	1.224	1.6	361	1697.42
5	1.857	2.0	345	1721.78

7. OBSERVATIONS

The following important findings were noted for the developed inventory model, where demand depends on the amount of stock on hand and there are retroactive holding costs:

Demand Rate Dependency on Inventory Level

The main assumption of this model is that the demand rate is linear with the inventory level. This indicates that as product levels rise, so will the demand rate, as would be expected in a real-world situation where the availability of more product could result in higher sales. In real-world supply chains, availability and visibility play a key role in influencing sales, while marketing strategies rely on inventory.

Impact of Holding Cost on Inventory Policy

The new feature that was added to the model was the presence of a variable holding cost assumed to be an increasing step function of storage time. The holding cost depends on the time in storage, and is not constant for the entire inventory cycle. This mirrors real life situations such as deteriorating products or perishable goods, the longer the goods remain in store the more costly the product becomes as a result of spoilage, depreciation and limited space. This cost of holding the variable is very important in calculating the best order quantity and cycle time.

Shortage and Backordering

Shortages (if any) in the model are assumed to be backlogged, that is, carried over from one order to the next. The downside of this backorder system is that it requires flexibility in the supply chain, but the advantage is that it allows it to be responsive to fluctuations in demand. The back order costs are not specifically identified in the observations, but they can be considered as under the Total Inventory Cost (TIC) that encompasses ordering costs, holding costs and shortage costs. Shortages are incorporated into the model allowing for a more realistic representation of many industries where shortages are an inevitability and are manageable.

Cycle Time and Optimal Order Quantity Decrease with Higher Holding Costs

From the calculations, it can be seen that the value of holding cost, and the optimal order quantity and time both reduce as the holding cost increases. This is a logical approach – companies have an incentive to place smaller orders more often because of the cost to store for a longer period. The following pattern was noticed from the data:

- The longer the cycle time (T), the lower the holding cost, per unit, per time unit. This is because the longer the goods are in storage, the more expensive it becomes, making it easier to keep replenishing the stock.

- Similarly, the optimal order quantity (Q) decreases. This is probably due to the fact that the order volume is smaller, meaning that there is less inventory to be held, and thus less of the related inventory holding cost.

Ordering Cost and Total Inventory Cost (TIC)

The ordering cost in the model is assumed to be fixed per ordering cycle and is considered to represent the typical real-life situation where ordering costs are constant, irrespective of the order size (e.g. transportation cost, handling cost, administrative costs, etc.). As the holding cost goes up, however, the overall Total Inventory Cost (TIC) will increase, even though the optimal order quantity will minimize the TIC as per the model. Finding an optimum ordering policy that balances ordering cost, holding cost and shortage cost, the model can actually be used to determine the optimum quantity and optimum cycle time and can effectively minimize the total cost depending on various factors including holding cost.

Retroactive Increase of Holding Cost

This is a model that takes into account retroactive holding costs, one of its unique characteristics. In this situation the holding cost charged for each period of storage is dependent on the cost incurred during the preceding period. That is, when the storage period is longer than originally thought, the holding cost rises and this rise is retroactive to all earlier periods. The retroactive mechanism serves important purposes in products whose value decreases as they are stored (such as the perishable items, electronics etc.). This hindsight feedback then assists businesses to decide on order quantities and timing, thereby accelerating inventory turnover.

Effectiveness of Linear Demand Function

Assuming that the demand is linear with the inventory level is effective for capturing stock dependent demand; however, it may not provide a good model for more complicated

relationships between supply and demand, including non-linear and stochastic demand. Other factors such as seasonal trends, promotions, or market conditions can affect the demand in practical applications. For the purposes of this model, however, the linear function offers a simple and straightforward method to understanding the effect of inventory on demand and how the company can optimize inventory policy.

Backlog Management and Its Costs

The model assumes that shortages lead to backlog orders, which in turn are met when an order is received. This approach will reduce the immediate disruption caused by inventory shortages, but it will also come with the cost of holding back and missing sales, as well as customer frustration with long wait times. These costs are not explicitly computed by this model, but rather are included as part of the Total Inventory Cost. A level of buffer stock can help control the effect of backordering, but this will also raise holding costs, which will result in a trade-off.

Practical Implications for Inventory Management

This model is very useful in cases of businesses having products with deteriorating or perishable products, where holding costs are not fixed but rather rising over time. In such cases, businesses need to balance the cost of holding excess inventory with the risk of stockouts. The model includes variable holding costs and demand that is dependent on inventory, providing valuable insights into how businesses can optimize their inventory systems. These results indicate that businesses should make dynamic changes to their stock inventory policies depending on the holding costs and be ready to place orders more often to prevent them from incurring too much holding costs.

Computational and Real-World Applicability

The computational techniques employed to compute the optimal order quantity and order cycle in this model work well and can be applied with commonly available inventory management software. The model is able to be applied to a variety of industries, ranging from high-speed consumer goods to product industries with perishable or sensitive products, thanks to the integration of the different mechanisms of demand, holding cost and backorder. The model can also be extended to include additional complexities such as stochastic demand, multiple items or multi-echelon supply chains.

8. CONCLUSIONS

This paper has introduced an advanced inventory model in which the demand is stock dependent, the holding cost is variable and shortages are considered, making the inventory model more realistic and nuanced. The model in particular incorporates the idea of retroactive rise in holding costs, which are applied in steps as inventory is stored for longer and longer time to reflect the nature of industries with perishable or deteriorating products. The one of the core conclusions of the model is that there is a trade-off between holding costs and both the optimal order quantity and the cycle time. This finding emphasizes the need for reducing storage duration to prevent the spiraling of costs that can have a substantial impact on profitability. Furthermore, the model includes the concept of backordering, which is suitable for companies that would like to avoid losing sales or customers' dissatisfaction even during the periods of stockout, albeit they might occasionally experience shortages. Because the model represents demand as a function of the level of inventory, it also is able to model an important aspect of the real world: when products are more available, sales are stimulated. This is an aspect that many businesses see happening, but may not be modeled accurately through conventional inventory models. Moreover, the incorporation of such variable holding costs, especially those

that increase retroactively, also contributes depth and realism to the model and makes it highly applicable to the industries where the goods have some time-related cost to its storage, such as fresh produce, pharmaceuticals or electronic components. Overall, this inventory system model provides useful insights for optimizing both ordering decisions and cycle time, and also helps in reducing the total cost while doing a good job when there are shortages and inventory replenishment. The versatility of the model allows it to be tailored to different industries and inventories, making it a valuable tool for businesses aiming to improve their inventory management. The results and the framework described in this paper provide a solid basis for future research and practical applications in real-world inventory management situations to help companies make the complicated decisions between order size, stock holding costs and demand fulfillment.

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