Shodhbodhalaya: An International Peer reviewed Multidisciplinary Journal

Volume 4 | Issue 2 | ISSN: 2584-1807 | (https://shodhbodh.com/)

Analysis of Various Queueing Models Using Fuzzy Logic

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Abstract

Queueing models are widely used to study waiting-line systems in service, transportation, production, communication, and industrial environments. Classical queueing theory usually assumes that arrival rates, service rates, and batch sizes are known exactly. In many real-life situations, however, these parameters are uncertain and are better represented through linguistic or fuzzy values. This paper analyzes bulk-arrival multi-server queueing models under triangular and trapezoidal fuzzy environments. The α-cut method and the DSW algorithm are used to obtain fuzzy performance measures, including the expected number of customers in the queue, expected number of customers in the system, mean waiting time in the queue, and mean waiting time in the system. A numerical example based on a toll-tax system is used, where vehicles arrive in bulk and are served through three toll counters. The results show that triangular fuzzy numbers provide narrower performance intervals, while trapezoidal fuzzy numbers produce wider uncertainty ranges. The comparison helps identify how different fuzzy representations affect queueing performance under uncertain arrival and service conditions.

Keywords: queueing theory, fuzzy logic, bulk arrival, triangular fuzzy number, trapezoidal fuzzy number, α-cut, DSW algorithm, multi-server queue, performance measures.

1. Introduction

Queueing theory consists of mathematical models used to study waiting-line systems. Such systems appear in many practical situations, including toll plazas, banks, hospitals, production units, service counters, transport systems, and communication networks. In these systems, customers, vehicles, jobs, or materials arrive for service and may have to wait when all service facilities are busy. In classical queueing theory, arrival time and service time are generally represented by probability distributions. However, in many real-life cases, the available information is not exact. Arrival and service patterns may be described by linguistic terms such as "slow," "moderate," "rapid," "low," or "high." In such cases, fuzzy queueing models are more realistic than crisp queueing models because they allow uncertainty and vagueness to be included in mathematical analysis.

The concept of fuzzy sets was introduced by Zadeh[23]. Later, Zadeh also developed possibility theory, which became useful for representing uncertain information[24]. Dubois and Prade[25], Kaufmann[26], Kaufmann and Gupta[27], and Zimmermann[28] further developed fuzzy mathematics and fuzzy systems. In queueing theory, Buckley studied elementary queueing systems based on possibility theory[29]. Prade discussed fuzzy or possibilistic models for queueing systems[30]. Li and Lee analyzed fuzzy queues using fuzzy arrival and service concepts[31]. Negi and Lee applied α-cut and simulation approaches to fuzzy queues[32]. Kao, Li, and Chen used parametric programming for fuzzy queue analysis[33], while Chen studied fuzzy queues with bulk service using a parametric nonlinear programming approach[34].

This paper studies bulk-arrival multi-server queueing models under fuzzy conditions. Two types of fuzzy numbers are used: triangular fuzzy numbers and trapezoidal fuzzy numbers. The purpose is to calculate and compare fuzzy performance measures for both models using the α-cut method and the DSW algorithm.

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[33] Kao C, Li CC, Chen SP. Parametric programming to the analysis of fuzzy queues. Fuzzy Sets and Systems. 1999;107(1):93-100.
[34] Chen SP. Parametric nonlinear programming approach to fuzzy queues with bulk service. European Journal of Operational Research. 2005;163(2):434-444.

2. Mathematical Preliminaries

2.1 α-Cut

Let Abe a fuzzy set defined on X. For any $\alpha \in [0,1]$, the α-cut of fuzzy set $A$is defined as:

$$A_\alpha = \{ x \mid \mu_A(x) \geq \alpha,\ x \in Z \}$$

It can also be written as:

$$A_\alpha = [L_A(\alpha), U_A(\alpha)]$$

where $A_\alpha$is a non-empty bounded interval contained in Z. Here, $L_A(\alpha)$and $U_A(\alpha)$represent the lower and upper bounds of the α-cut of A, respectively.

2.2 Triangular Fuzzy Number

A triangular fuzzy number is represented by:

$$D = (u_1, u_2, u_3)$$

where $u_1$is the lower value, $u_2$is the most likely value, and $u_3$is the upper value. Its membership function is:

$$\mu_D(x) = \begin{cases} 0, & x \leq u_1 \\ \dfrac{x - u_1}{u_2 - u_1}, & u_1 \leq x \leq u_2 \\ 1, & x = u_2 \\ \dfrac{u_3 - x}{u_3 - u_2}, & u_2 \leq x \leq u_3 \\ 0, & \text{otherwise} \end{cases}$$

The α-cut form of a triangular fuzzy number is:

$$D_\alpha = [u_1 + \alpha(u_2 - u_1), u_3 - \alpha(u_3 - u_2)]$$

2.3 Trapezoidal Fuzzy Number

A trapezoidal fuzzy number is represented by:

$$\widetilde{U} = (u_1, u_2, u_3, u_4)$$

where:

$$u_1 \le u_2 \le u_3 \le u_4$$

The membership function is:

$$\mu_{\widetilde{U}}(x) = \begin{cases} 0, & x < u_1 \\ \dfrac{x - u_1}{u_2 - u_1}, & u_1 \le x \le u_2 \\ 1, & u_2 \le x \le u_3 \\ \dfrac{x - u_4}{u_3 - u_4}, & u_3 \le x \le u_4 \\ 0, & \text{otherwise} \end{cases}$$

The α-cut form of a trapezoidal fuzzy number is:

$$\widetilde{U}_\alpha = [u_1 + \alpha(u_2 - u_1), u_4 - \alpha(u_4 - u_3)]$$

3. Fuzzy Interval Arithmetic

Let $Z_1$and Z_2be two interval numbers:

$$Z_1 = [x, y]$$
$$Z_2 = [u, v]$$

The general interval operation is:

$$Z_1 * Z_2 = [x, y] * [u, v]$$

where:

$$* = \{+, -, \times, \div\}$$

The basic operations are:

$$Z_1 + Z_2 = [x + u,\ y + v]$$
$$Z_1 - Z_2 = [x - v,\ y - u]$$
$$Z_1 \times Z_2 = [\min(xu, xv, yu, yv), \max(xu, xv, yu, yv)]$$
$$Z_1 \div Z_2 = [x, y]\left[\frac{1}{v}, \frac{1}{u}\right]$$

provided that:

$$u, v \neq 0$$

For scalar multiplication:

$$\alpha[x, y] = \begin{cases} [\alpha x, \alpha y], & \alpha > 0 \\ [\alpha y, \alpha x], & \alpha < 0 \end{cases}$$

4. DSW Algorithm

The DSW algorithm is used to obtain the membership functions of fuzzy performance measures. It works with intervals at different α-cut levels and applies standard interval arithmetic[35].

The steps are:

Step 1: Select a value of αin the range $[0, 1]$.

Step 2: Find the input membership-function intervals corresponding to the selected α-cut.

Step 3: Determine the output interval using interval arithmetic.

Step 4: Repeat the same process for different α-cut values to obtain the complete fuzzy solution.

5. Performance Measures of the $FM^B/FM/C$Queue

For the bulk-arrival multi-server fuzzy queueing model, the following performance measures are used.

5.1 Mean Number of Customers in the Queue

$$L_q^\alpha = \frac{B_{a1}\lambda^\alpha \mu^\alpha \left(\frac{B_{a1}\lambda^\alpha}{\mu^\alpha}\right)^c}{(c-1)!\,(c\mu^\alpha - B_{a1}\lambda^\alpha)^2} P_0$$

5.2 Mean Number of Customers in the System

$$L_s^\alpha = \frac{B_{a1}\lambda^\alpha \mu^\alpha \left(\frac{B_{a1}\lambda^\alpha}{\mu^\alpha}\right)^c}{(c-1)!\,(c\mu^\alpha - B_{a1}\lambda^\alpha)^2} P_0 + \frac{B_{a1}\lambda^\alpha}{\mu^\alpha}$$

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[35] Gross D, Shortle JF, Thompson JM, Harris CM. Fundamentals of Queueing Theory. 4th ed. Hoboken: John Wiley & Sons; 2008.

5.3 Mean Waiting Time in the Queue

$$W_q^{\alpha} = \frac{\mu^{\alpha}\left(\frac{B_{a1}\lambda^{\alpha}}{\mu^{\alpha}}\right)^{c}}{(c-1)!\,(c\mu^{\alpha} - B_{a1}\lambda^{\alpha})^{2}} P_0$$

5.4 Mean Waiting Time in the System

$$W_s^{\alpha} = \frac{\mu^{\alpha}\left(\frac{B_{a1}\lambda^{\alpha}}{\mu^{\alpha}}\right)^{c}}{(c-1)!\,(c\mu^{\alpha} - B_{a1}\lambda^{\alpha})^{2}} P_0 + \frac{1}{\mu^{\alpha}}$$

5.5 Value of P_0

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{\left(\frac{B_{a1}\lambda^{\alpha}}{\mu^{\alpha}}\right)^{n}}{n!} + \frac{\left(\frac{B_{a1}\lambda^{\alpha}}{\mu^{\alpha}}\right)^{c}}{c!} \cdot \frac{c\mu^{\alpha}}{c\mu^{\alpha} - B_{a1}\lambda^{\alpha}}}$$

6. Numerical Example

A real-life toll-tax example is considered. Vehicles such as cars, buses, and trucks arrive in bulk and in uncertain ways. The system has three toll counters on the Bhiwani to Rohtak route. The arrival rate and service rate are represented using fuzzy numbers.[36] The batch size of arriving vehicles is taken as:

$$B_{a1} = 3$$

[36] Kleinrock L. Queueing Systems, Volume I: Theory. New York: John Wiley & Sons; 1975.

The number of servers is:

$$c = 3$$

Two fuzzy models are analyzed:

1. Bulk-arrival queue with triangular fuzzy numbers.
2. Bulk-arrival queue with trapezoidal fuzzy numbers.

7. Bulk-Arrival Queue with Triangular Fuzzy Number

For the triangular fuzzy model, the arrival rate and service rate are:

$$\lambda = [8,9,10]$$
$$\mu = [12,13,14]$$

The α-cut values are:

$$\lambda^{\alpha} = [8 + \alpha,\ 10 - \alpha]$$
$$\mu^{\alpha} = [12 + \alpha,\ 14 - \alpha]$$

The performance measures obtained at different α-cut levels are shown in Table 1.

Table 1. Performance Measures for Triangular Fuzzy Number Model

α	L_q^{α}	L_s^{α}	W_q^{α}	W_s^{α}
0	[0.096, 12.16]	[1.806, 14.66]	[0.003, 0.506]	[0.071, 0.589]

0.2	[0.163, 7.090]	[1.943, 9.500]	[0.005, 0.288]	[0.077, 0.369]
0.4	[0.268, 4.310]	[2.118, 6.630]	[0.009, 0.171]	[0.083, 0.251]
0.6	[0.432, 2.677]	[2.352, 4.907]	[0.015, 0.103]	[0.089, 0.182]
0.8	[0.687, 1.703]	[2.687, 3.868]	[0.025, 0.064]	[0.100, 0.142]
1	[1.080, 1.080]	[3.158, 3.158]	[0.040, 0.040]	[0.120, 0.120]

From Table 1, the expected number of vehicles in the queue is [1.080, 1.080], and it is impossible for the value to fall outside [0.096, 12.16].

The expected number of vehicles in the system is [3.158, 3.158], and it is impossible for the value to fall outside [1.806, 14.66].

The average waiting time of vehicles in the queue is [0.040, 0.040], and it is impossible for the value to fall outside [0.003, 0.506].

The average waiting time of vehicles in the system is [0.120, 0.120], and it is impossible for the value to fall outside [0.071, 0.589].

8. Graphical Representation for Triangular Fuzzy Model

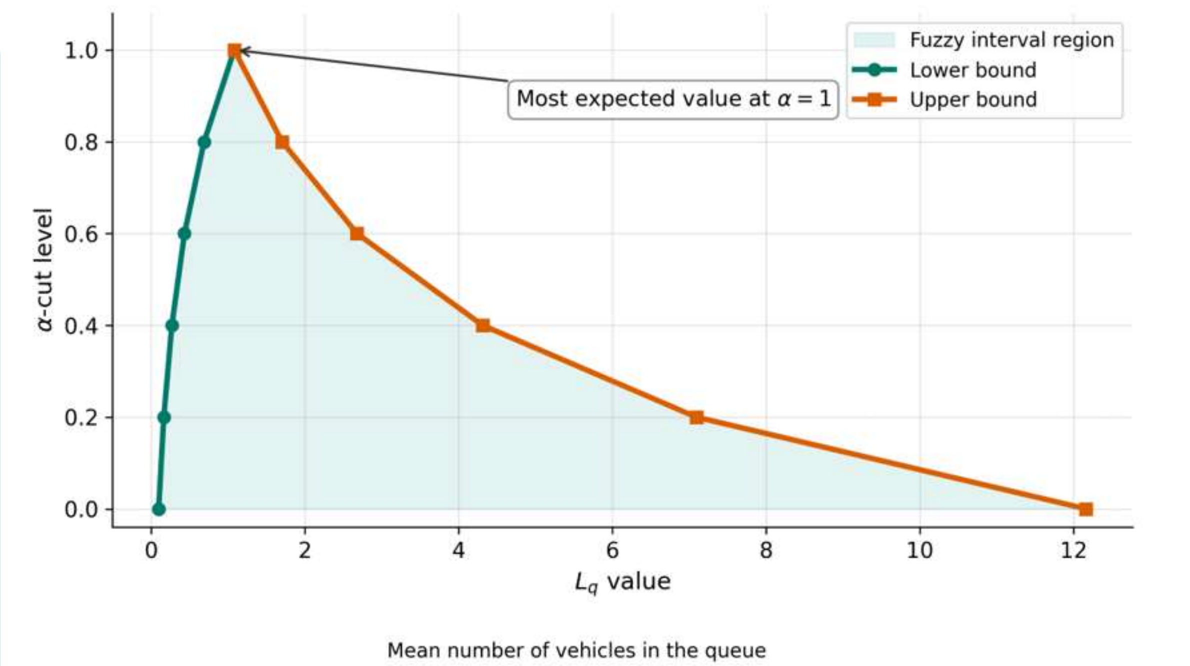

Figure 1. $L_q$for triangular fuzzy number model

This figure represents the average number of vehicles in the queue. The minimum value of the interval increases as αapproaches 1, while the maximum value decreases. At $\alpha = 1$, the result becomes the most certain value.

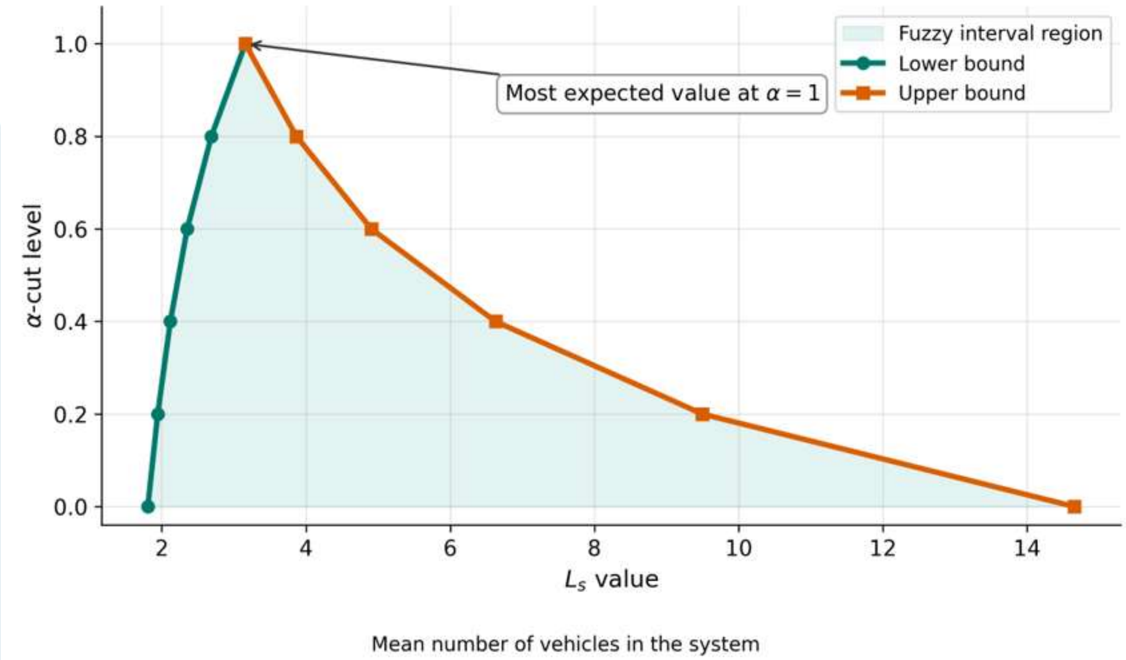

Figure 2. L_s **for triangular fuzzy number model**

This figure represents the average number of vehicles in the system. It includes both vehicles waiting in the queue and vehicles receiving service.

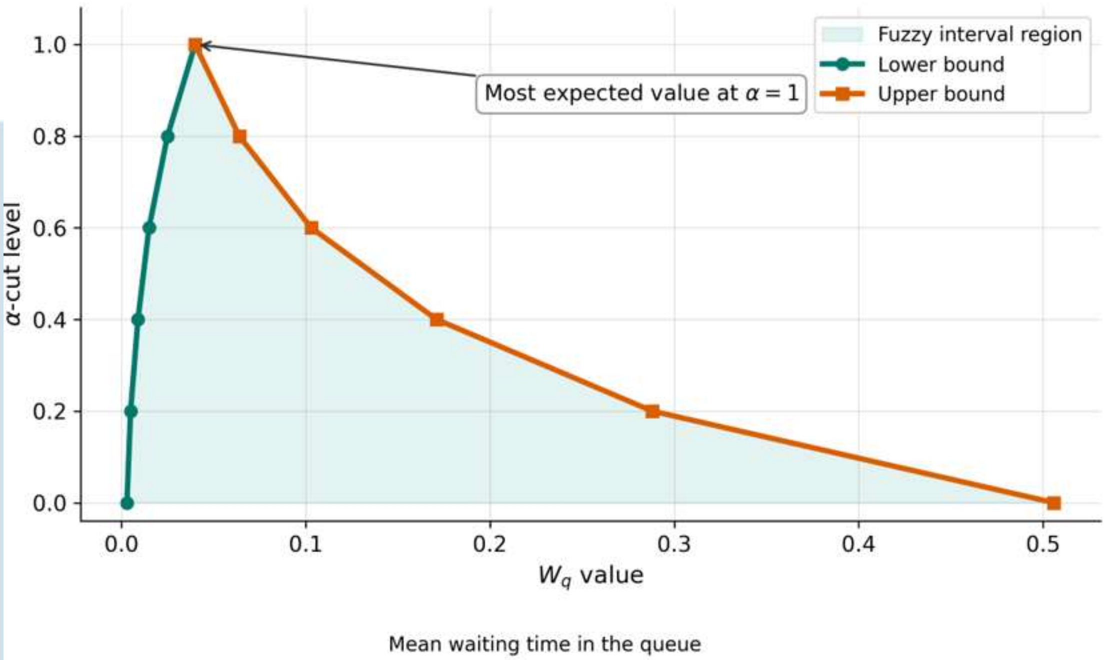

Figure 3. W_q for triangular fuzzy number model

This figure represents the average waiting time of vehicles in the queue. As α increases, the uncertainty interval becomes narrower.

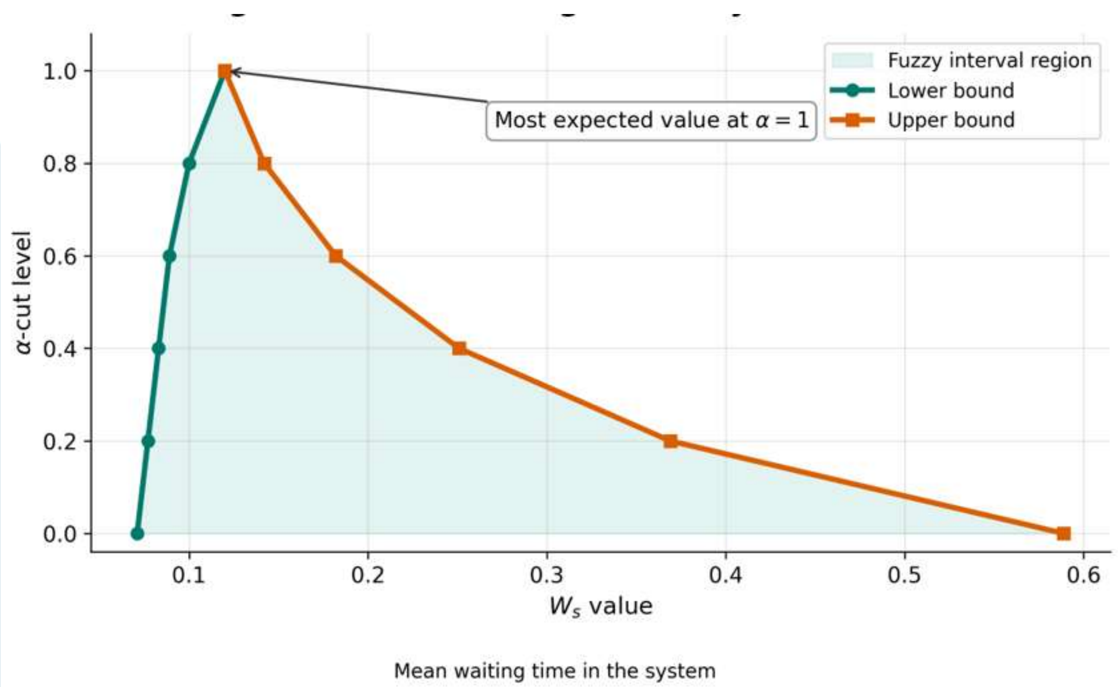

Figure 4. W_s **for triangular fuzzy number model**

This figure represents the average waiting time of vehicles in the system. The value at $\alpha = 1$is the most expected value.

9. Bulk-Arrival Queue with Trapezoidal Fuzzy Number

For the trapezoidal fuzzy model, the arrival rate and service rate are:

$$\tilde{\lambda} = [8,9,10,11]$$
$$\tilde{\mu} = [12,13,14,15]$$

The α-cut values are:

$$\lambda^{\alpha} = [8 + \alpha,\ 11 - \alpha]$$
$$\mu^{\alpha} = [12 + \alpha,\ 15 - \alpha]$$

The number of servers is:

$$c = 3$$

The batch size is:

$$B_{a1} = 3$$

The performance measures are shown in Table 2.

Table 2. Performance Measures for Trapezoidal Fuzzy Number Model

α	L_q^α	L_s^α	W_q^α	W_s^α
0	[0.033, 75.47]	[1.633, 78.22]	[0.001, 3.144]	[0.067, 3.227]
0.2	[0.041, 33.37]	[1.701, 36.02]	[0.0012, 1.356]	[0.068, 1.437]
0.4	[0.083, 17.31]	[1.809, 19.87]	[0.0026, 0.676]	[0.071, 0.756]
0.6	[0.143, 9.824]	[1.933, 12.304]	[0.0045, 0.38]	[0.073, 0.459]
0.8	[0.228, 5.944]	[2.078, 8.334]	[0.0074, 0.225]	[0.077, 0.303]
1	[0.307, 3.704]	[2.207, 6.000]	[0.0102, 0.137]	[0.081, 0.213]

From Table 2, the expected number of vehicles in the queue is $[0.307, 3.704]$, and it is impossible for the value to fall outside $[0.033, 75.47]$.

The expected number of vehicles in the system is $[2.207, 6.000]$, and it is impossible for the value to fall outside $[1.633, 78.22]$.

The average waiting time of vehicles in the queue is $[0.0102, 0.137]$, and it is impossible for the value to fall outside $[0.001, 3.144]$.

The average waiting time of vehicles in the system is $[0.081, 0.213]$, and it is impossible for the value to fall outside $[0.067, 3.227]$.

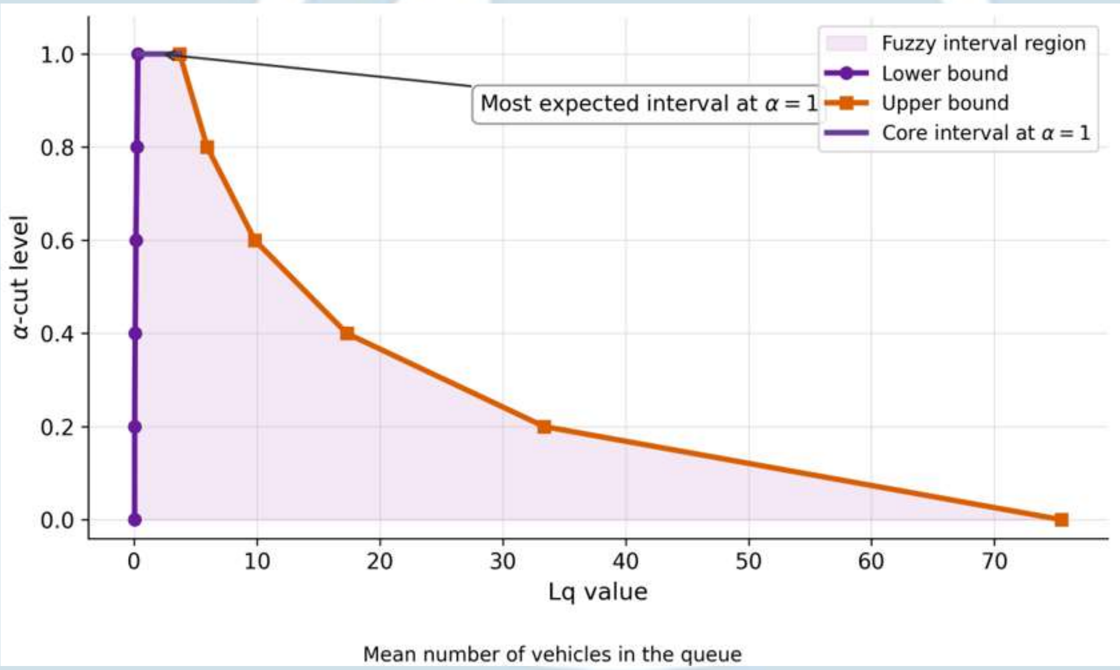

Figure 5. L_q for trapezoidal fuzzy number model

This figure shows that the minimum value of the queue interval increases with α, while the maximum value decreases. The expected mean length at $\alpha = 1$is $[0.307, 3.704]$.

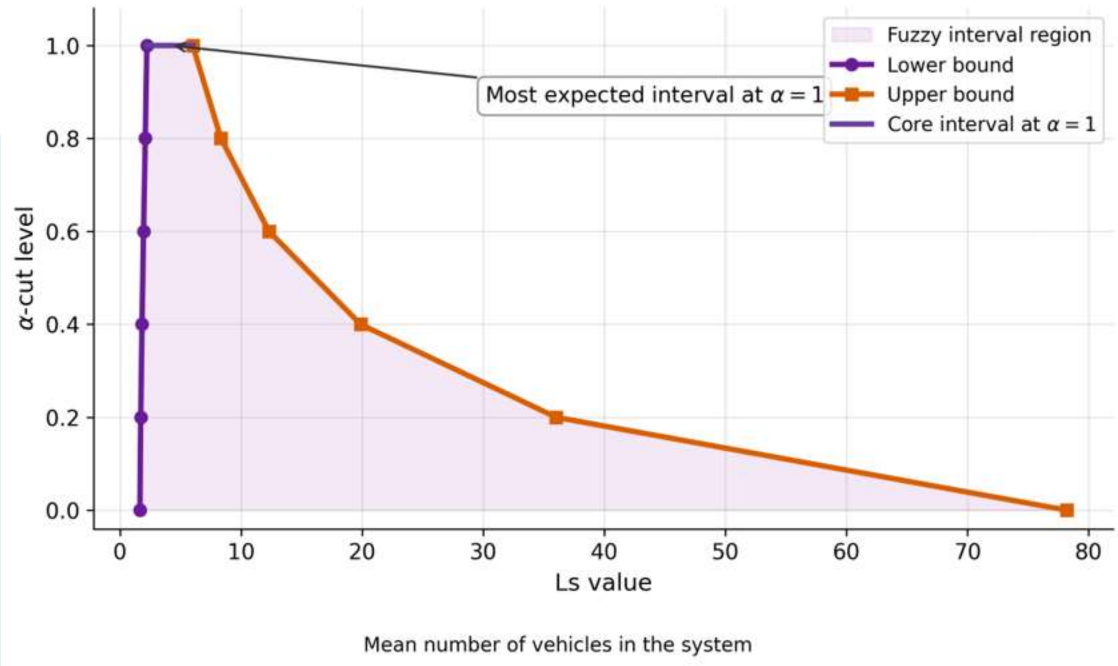

Figure 6. $L_s$for trapezoidal fuzzy number model

This figure represents the expected number of vehicles in the system. The most expected interval at $\alpha = 1$is $[2.207, 6.000]$.

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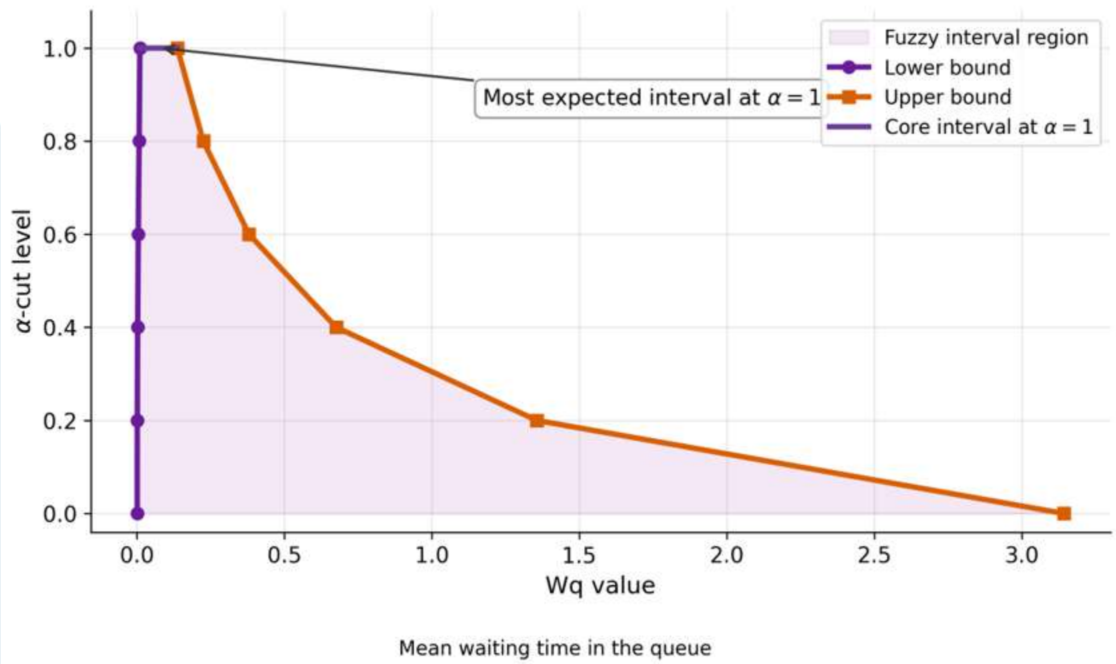

Figure 7. W_q **for trapezoidal fuzzy number model**

This figure represents the mean waiting time in the queue. The most expected interval at $\alpha = 1$is $[0.0102, 0.137]$.

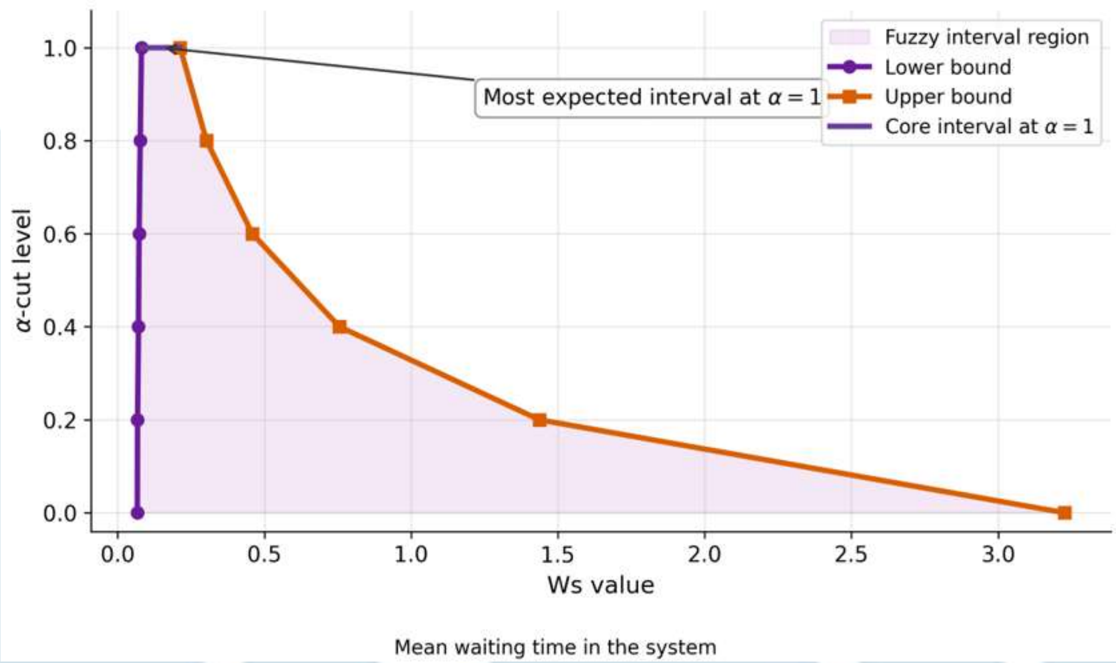

Figure 8. W_s for trapezoidal fuzzy number model

This figure represents the mean waiting time in the system. The most expected interval at $\alpha = 1$is $[0.081, 0.213]$.

11. Comparison of Triangular and Trapezoidal Fuzzy Models

This section compares the triangular fuzzy number model and trapezoidal fuzzy number model for the same bulk-arrival toll-tax queueing system.

Table 3. Minimum Values of L_q

α	Triangular fuzzy number	Trapezoidal fuzzy number
0	0.096	0.033

0.2	0.163	0.041
0.4	0.268	0.083
0.6	0.432	0.143
0.8	0.687	0.228
1	1.080	0.307

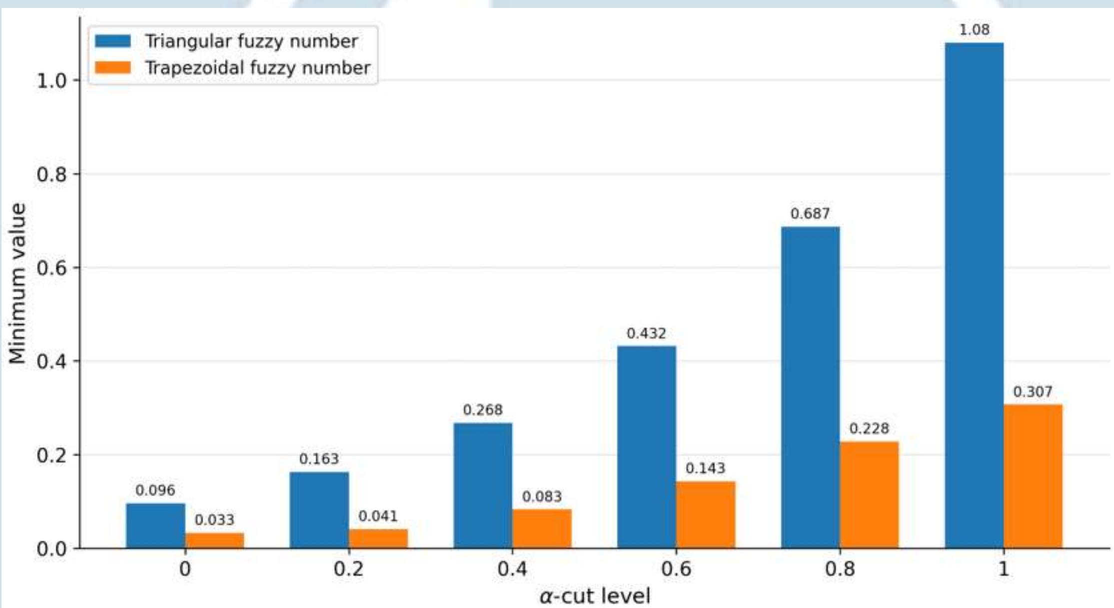

Figure 9. Minimum values of L_q with respect to triangular and trapezoidal fuzzy numbers

The minimum value of L_q is lower for the trapezoidal fuzzy model than for the triangular fuzzy model. In both models, the minimum value increases as α increases.

Table 4. Maximum Values of L_q

α	Triangular fuzzy number	Trapezoidal fuzzy number
0	12.16	75.47
0.2	7.09	33.37
0.4	4.31	17.31
0.6	2.677	9.824
0.8	1.703	5.944
1	1.080	3.704

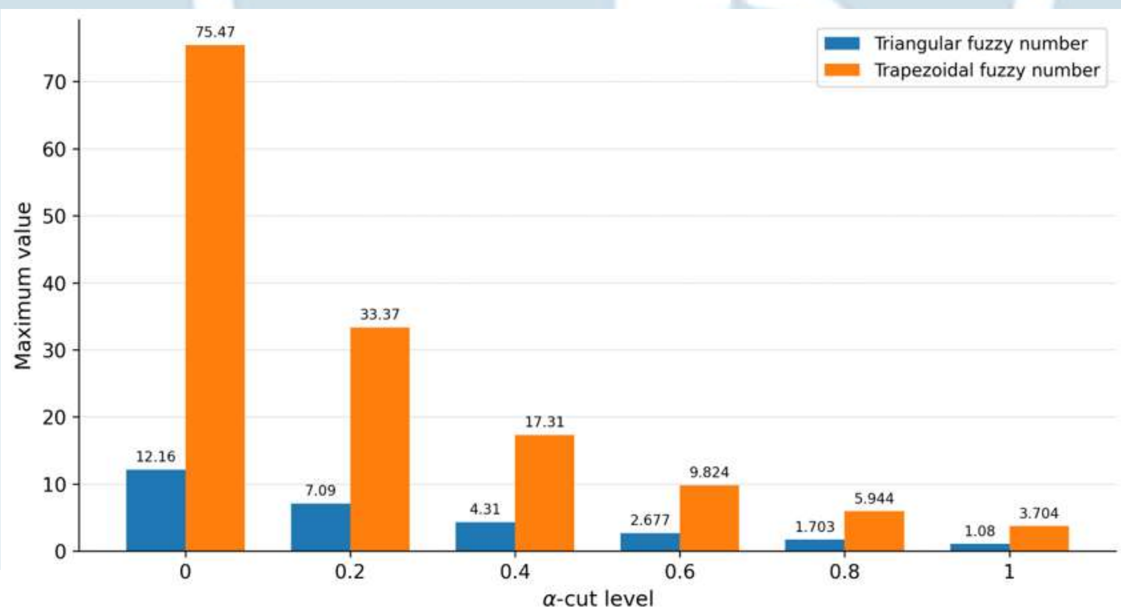

Figure 10. Maximum values of L_q with respect to triangular and trapezoidal fuzzy numbers

The maximum value of L_q decreases as α increases. The trapezoidal fuzzy number model gives higher maximum values than the triangular fuzzy number model.

Table 5. Minimum Values of L_s

α	**Triangular fuzzy number**	**Trapezoidal fuzzy number**
0	1.805	1.633
0.2	1.943	1.701
0.4	2.118	1.809
0.6	2.352	1.933
0.8	2.6877	2.078
1	3.158	2.207

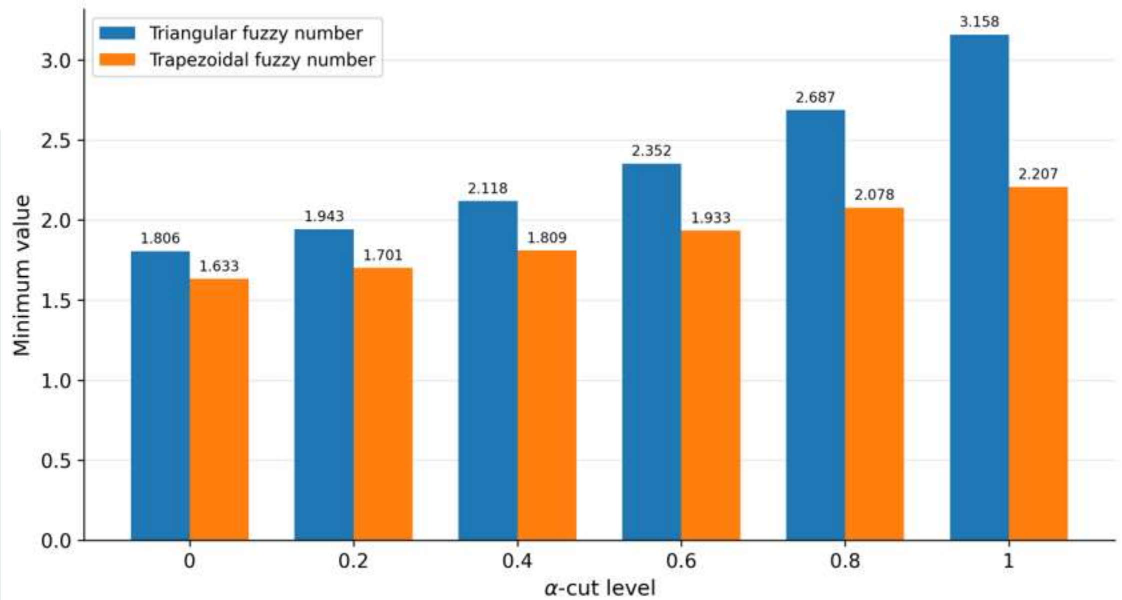

Figure 11. Minimum values of L_s with respect to triangular and trapezoidal fuzzy numbers

The minimum value of the expected number of vehicles in the system is higher in the triangular fuzzy model than in the trapezoidal fuzzy model.

Table 6. Maximum Values of L_s

α	**Triangular fuzzy number**	**Trapezoidal fuzzy number**
0	14.66	78.22
0.2	9.50	36.02
0.4	6.63	19.87
0.6	4.907	12.304

0.8	3.868	8.334
1	3.158	6.000

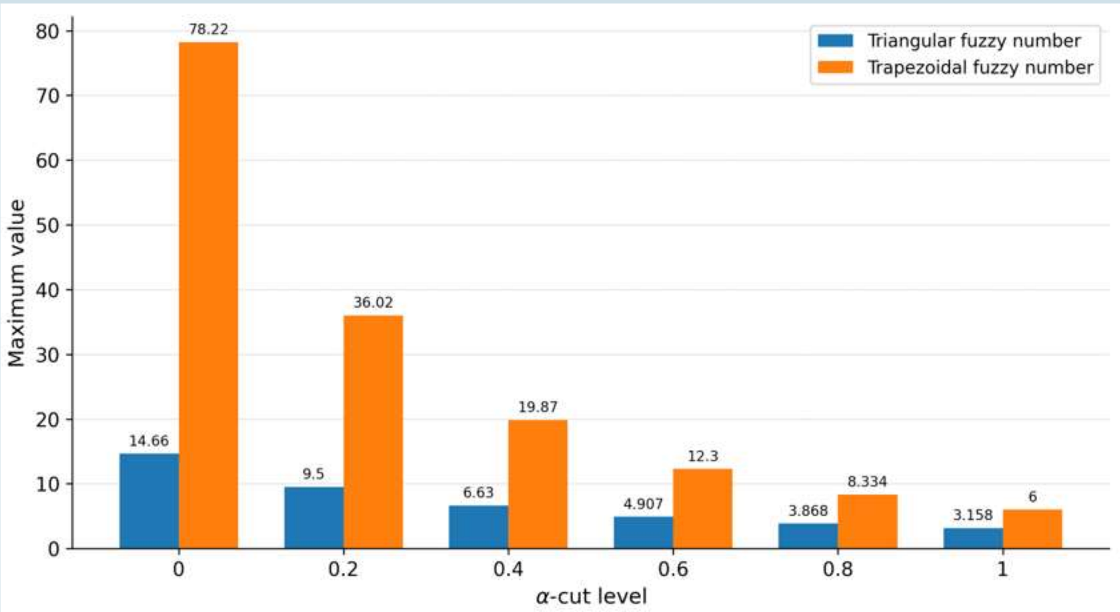

Figure 12. Maximum values of $L_s$with respect to triangular and trapezoidal fuzzy numbers

The maximum value of $L_s$is higher for the trapezoidal fuzzy model. This shows that the trapezoidal fuzzy interval is wider than the triangular fuzzy interval.

Table 7. Minimum Values of W_q

α	**Triangular fuzzy number**	**Trapezoidal fuzzy number**
0	0.003	0.001
0.2	0.005	0.0012

0.4	0.009	0.0026
0.6	0.015	0.0045
0.8	0.025	0.0074
1	0.040	0.0102

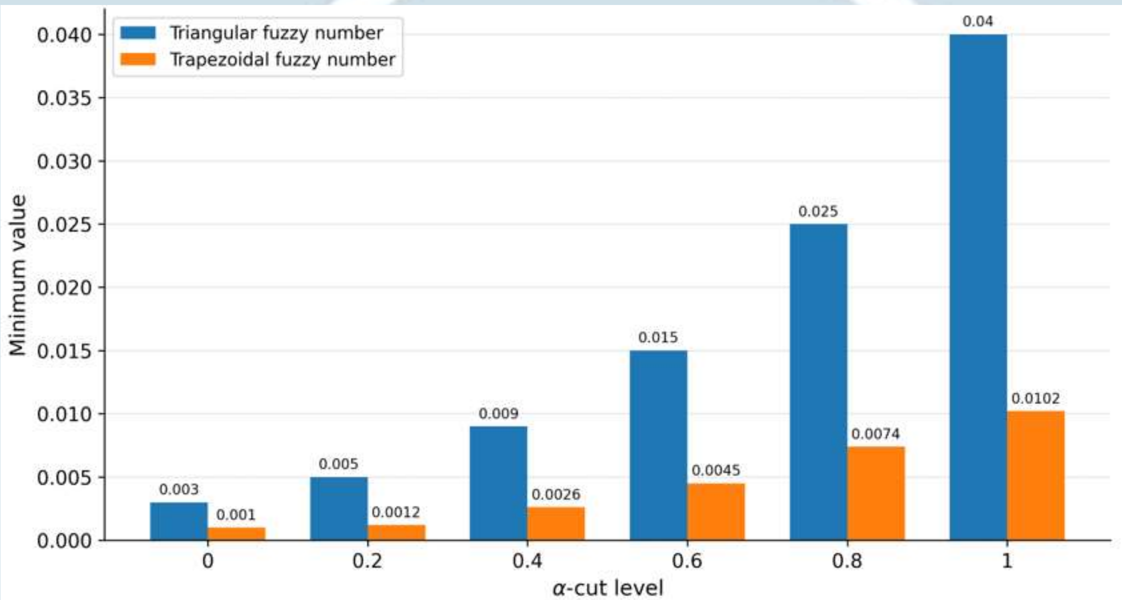

Figure 13. Minimum values of W_q with respect to triangular and trapezoidal fuzzy numbers

The minimum waiting time in the queue is higher in the triangular fuzzy model than in the trapezoidal fuzzy model.

Table 8. Maximum Values of W_q

α	**Triangular fuzzy number**	**Trapezoidal fuzzy number**

0	0.506	3.144
0.2	0.288	1.356
0.4	0.171	0.676
0.6	0.103	0.380
0.8	0.064	0.225
1	0.040	0.137

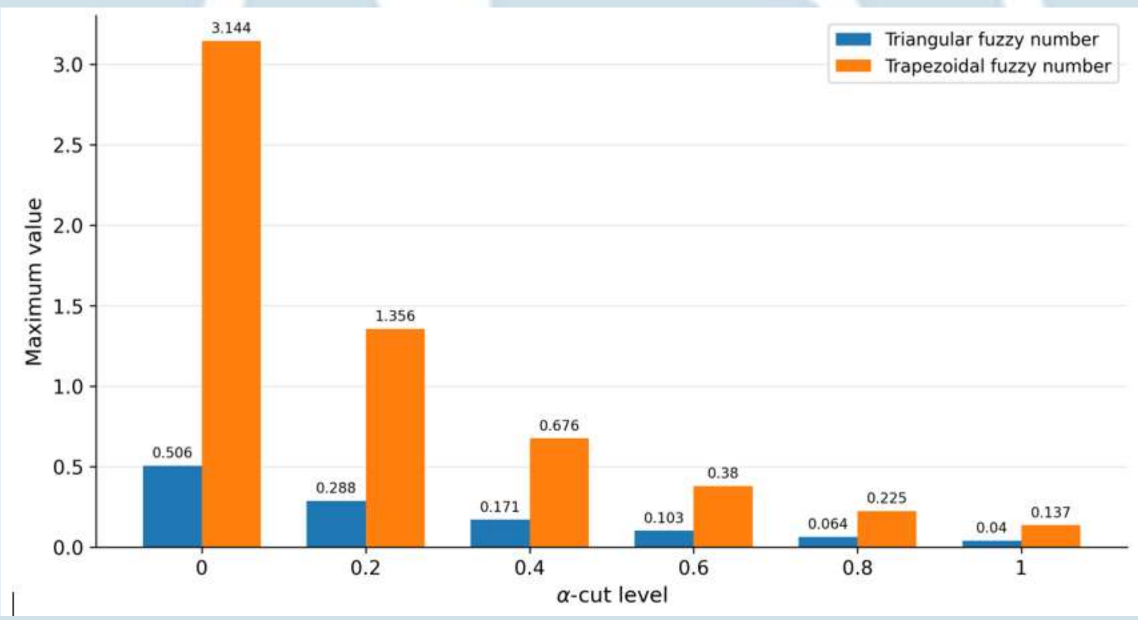

Figure 14. Maximum values of W_q with respect to triangular and trapezoidal fuzzy numbers

The maximum waiting time in the queue is greater in the trapezoidal fuzzy model. This confirms that the trapezoidal fuzzy number gives a wider range of uncertainty.

Table 9. Minimum Values of W_s

α	**Triangular fuzzy number**	**Trapezoidal fuzzy number**
0	0.0714	0.067
0.2	0.077	0.068
0.4	0.0825	0.071
0.6	0.089	0.073
0.8	0.100	0.077
1	0.120	0.081

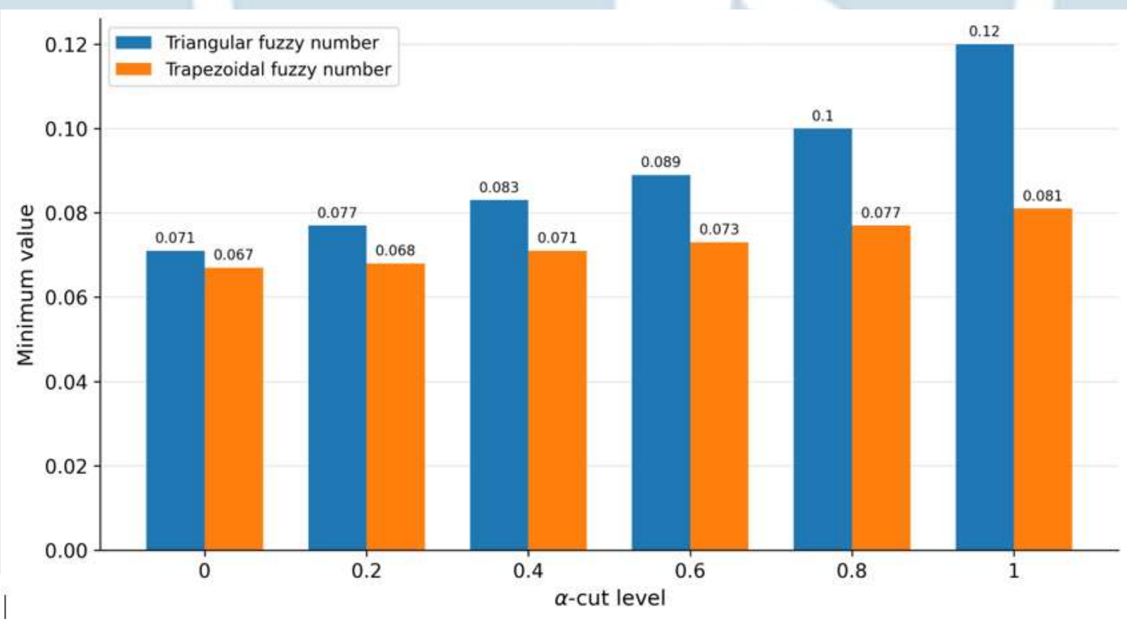

Figure 15. Minimum values of $W_s$with respect to triangular and trapezoidal fuzzy numbers

The minimum waiting time in the system is lower in the trapezoidal fuzzy model than in the triangular fuzzy model.

Table 10. Maximum Values of W_s

α	Triangular fuzzy number	Trapezoidal fuzzy number
0	0.5896	3.227
0.2	0.369	1.437
0.4	0.251	0.756
0.6	0.182	0.459
0.8	0.142	0.303
1	0.120	0.213

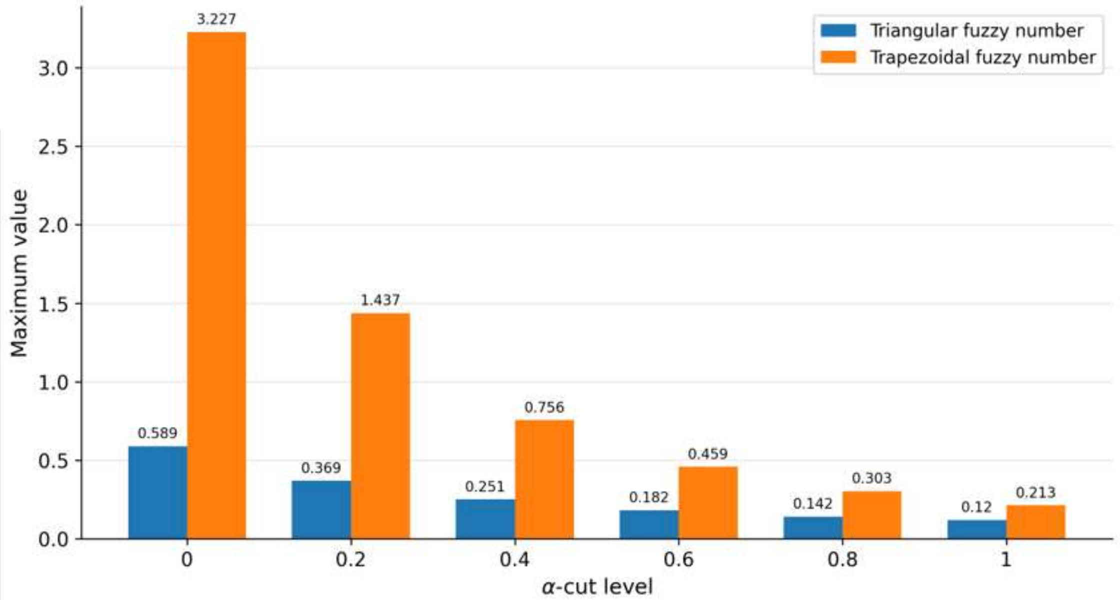

Figure 16. Maximum values of W_s with respect to triangular and trapezoidal fuzzy numbers

The maximum waiting time in the system is higher in the trapezoidal fuzzy model. Therefore, the trapezoidal model produces a wider interval than the triangular model.

12. Discussion

The numerical results show that both triangular and trapezoidal fuzzy numbers can be used to model uncertainty in bulk-arrival queueing systems. In the triangular fuzzy model, the interval becomes narrow at $\alpha = 1$, giving a single most expected value.[37] In the trapezoidal fuzzy model, the interval remains wider because the membership function has a flat region where several values may be considered fully possible. For the toll- example, the trapezoidal fuzzy model gives larger upper bounds for L_q, L_s, W_q, and W_s.[38] This means that the trapezoidal model captures a greater level of uncertainty. The triangular model gives more precise results, while the trapezoidal model gives more flexible and conservative results.[39] The comparison shows that the choice of fuzzy number affects the final performance measures. If the system information is

[37] Medhi J. Stochastic Models in Queueing Theory. 2nd ed. Amsterdam: Academic Press; 2003.
[38] Taha HA. Operations Research: An Introduction. 10th ed. Boston: Pearson; 2017.
[39] Ross SM. Introduction to Probability Models. 11th ed. Amsterdam: Academic Press; 2014.

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relatively clear, triangular fuzzy numbers are suitable.[40] If the uncertainty is wider and several values are equally possible, trapezoidal fuzzy numbers are more appropriate.

13. Conclusion

This paper analyzed bulk-arrival multi-server queueing models using fuzzy logic. The study considered two fuzzy environments: triangular fuzzy numbers and trapezoidal fuzzy numbers. The α-cut method and DSW algorithm were applied to calculate fuzzy performance measures. The numerical example was based on a toll-tax system with vehicles arriving in bulk and three toll counters serving them. The results showed that the triangular fuzzy number model gives narrower intervals, while the trapezoidal fuzzy number model gives wider intervals. The expected number of vehicles in the queue, expected number of vehicles in the system, average waiting time in the queue, and average waiting time in the system were calculated for different α-cut levels. The comparison confirms that trapezoidal fuzzy numbers represent greater uncertainty than triangular fuzzy numbers. Therefore, the trapezoidal fuzzy model is useful when the decision maker wants a broader and more cautious range of queueing performance. The triangular fuzzy model is useful when the system parameters are more concentrated around a most likely value. Overall, fuzzy queueing models are more realistic than crisp queueing models when arrival rates, service rates, and batch sizes are uncertain. Such models can help toll plazas, industries, service centers, and transportation systems decide whether more or fewer service channels are required.

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