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Mathematical Modeling on Sustainable Supply Chain Management with Controllable Carbon Emission

Researcher: Hardeep Ohlan[1]
Guide Name: Dr. Jagatveer Singh[2]
[1]Mail: ohlanhardeep@icloud.com

Abstract

To achieve a sustainable supply chain, a mathematically-based inventory model is proposed by incorporating green products, investment in conservation, and the impact of carbon tax policies, and demand of the product is conceived in terms of market potential, pricing policies, and environmentally sustainable consumption of the product. The model is general enough to recognise the perishability of items, use the investment in preservation technology variable to balance the loss rate and environmental cost, recognise conditions under which shortages are expected in the horizon period and allow the backlogging of items in part, which helps in real-world application. The objective of the proposed inventory systems is to minimise the total cost of the system. The total cost includes the cost of ordering, brining, holding, purchasing, shortage, lost sales, preservation investments, and carbon emissions. The environmental sustainability, technologies for conservation, and timing of the shortage are the variables to be modelled. The mathematical model is based on the application of inventory differential equations, costs and optimization with first-order and second-order conditions. The inventory is the central focus of the paper. The variables are supply, demand and price of environmental sustainability and conservation technologies. The optimality of the model is proved by the calculation of the Hessian matrix, and the validity of the proposed model is illustrated by a numerical example. A sensitivity analysis is presented in the end to see the influence of the main parameters on the total cost. Furthermore, the study establishes that investments in preservation and cost of green products can lessen damage and carbon emissions. Besides this, the costs of ordering, holding, deterioration rate, cycle length and selling price are the main factors determining the optimal total cost. The research contributes to sustainable supply chain management by introducing a mathematical model

that factors in cost optimisation constraints, a carbon tax regulatory framework, green initiatives and preservation technology investments.

Keywords: Sustainable supply chain management; Mathematical modeling; Green inventory model; Carbon tax policy; Controllable carbon emission; Preservation investment.

1. Introduction

Improvements in supply chain management involve a shift from customary just in time delivery/cost cutting approaches to a modern focus on balancing sustainability with cost. Companies which sell environmentally sustainable products are said to be at a competitive advantage. Sustainable supply chains can improve a company's image and customer loyalty, but governments have also implemented tools such as carbon taxes and emission ceilings to encourage companies to support environmentally friendly policies without losing competitive advantage. Green supply chain inventory systems focus on reducing the greenhouse gas emissions and externalities created from the transportation and inventory holding activities.

Products that are wasteful and designed to break quickly end up in landfills, which generates carbon emissions. Energy saving technologies can help repair, maintain or refurbish products and reduce carbon emissions. Conservation methods, sustainable business models, and strategies from the circular economy are mitigations against the environmental impacts generated by product deterioration. Preservation inventory models use conservation methods to minimise deterioration and maximise profit. The method aims at finding the best investment in preservation technologies to maximise profit and minimise costs.

2. Related Literature

2.1 Sustainable supply chain network design and carbon regulation

Sustainable supply chain management has received wide-ranging research attention as modern supply chains are expected to optimise total costs together with carbon emissions. Such mathematical models integrate the forward and reverse logistics, production planning, and supply chain network design of a product accounting for carbon emissions. Fuzzy strong optimisation models for forward and reverse logistics networks considering carbon dioxide emissions are shown by Shi et al. [1]. For industrial manufacturing, Li [3] focused on the decision making process for designing the sustainable supply chain networks in the context of minimising carbon emissions and observed the increasing emphasis on the move from customary objective functions to environmental objectives. This paper's perspective is in line with that of Kumar and Kumar [6], which formulated a low-carbon supply chain network problem under uncertainty. Their study highlights the need to include uncertainty when optimising low-carbon supply chain systems for efficient GHG emissions handling and supply chain cost management. Majumdar et al. [14] examined the design of decarbonised supply chain networks, including decisions related to facility location decision, mode of transport and topology of the supply chain under cap-and-trade regimes. Mehrbakhsh and Ghezavati [29] proposed a mathematical model for green supply chain design considering product recovery capacity and uncertainty in market demand, which highlights the role of recovery and reverse logistics decisions in green supply chain design.

Carbon emissions regulation instruments, including carbon tax, cap-and-trade, green subsidies and low carbon policy have drawn much attention. Kong and Yuan [4] studied how a capital-constrained manufacturer makes decisions on carbon mitigation actions in a cap-and-trade environment and analysed the effect of the capital constraint on carbon mitigation actions. A model for recycling, remanufacturing and technological innovations to achieve emissions reductions subject to the cap-and-trade level was developed by Wu and colleagues [5]. Zou and colleagues [8] studied a pricing model for low-carbon supply chains that accounted for corporate social responsibility within a carbon cap-and-trade setting. Zhang et al. [10] examined the emissions reduction of a two-tier supply chain under a Stackelberg game and carbon cap-and-trade regulation. Sun et al. [11] found the optimal pricing and low-carbon emission mitigation strategies that depended on a carbon cap-and-trade regulation with government subsidy policies in a closed-loop supply chain with consumers' carbon reduction behaviour. Xu et al. [12] investigated the equilibrium decisions of a low-carbon dual-channel supply chain considering product substitution with carbon tax policy and low-carbon subsidy policy. Miao et al. [20] studied the choices of emission reduction and pricing strategies under cap-and-trade and subsidy policies. Numerical simulations were conducted to illustrate the results. Carbon regulations influence the pricing, production practices, inventory management, recycling, and investment decisions to reduce carbon emissions within a firm's supply chain.

2.2 Coordination, pricing, and game-theoretic models for the low carbon supply chain

Many earlier studies have used game theory as well as supply chain coordination models to study the low carbon decisions of manufacturers, retailers, and other players in supply chains. Li et al. [9] construct stochastic differential game models to reduce carbon emissions in a four-tier supply chain considering benchmark low-carbon standards. Their observations indicate that reduction of emissions is a process that dynamically evolves through interaction. To analyse this dynamic reduction of emissions in low-carbon supply chains and multiple decision structures, he and his colleagues [23] applied differential game theory. Wang et al. [24] investigated carbon emission reduction decisions of the supply chain members in a cap-and-trade scheme via a differential game analysis. Lu et al. [28] formulated a multistage sustainable production-inventory model with carbon emission reduction and price-dependent demand under a Stackelberg game model. Pan and others [30] also developed a sustainable production-inventory model with a joint investment to reduce carbon emission by applying the Stackelberg game theory formulation. Mahato and others [7] developed a multistage inventory model for supply chains with controllable deterioration and imperfect production in presence of carbon emission constraint by using the Stackelberg game framework. Studies have shown that decentralised or coordinated supply chain structures can yield different optimal decisions regarding pricing, emissions reduction, production and inventory control.

Some other coordination mechanisms, such as the contract, vendor-managed inventory, revenue sharing, and cooperative investment, have also been studied in the context of low-carbon supply chains. For example, Mondal and Giri [22] studied sustainable supply chain with cap-and-trade policy and revenue sharing contract between the manufacturers and retailers. Their results showed that contractual coordination could bring about better economic performance and environmental performance. Astanti et al. [16] developed a low carbon supply chain model, under vendor managed inventory concept with carbon cap and trade regulation. The results of their analysis show an important reduction in carbon emissions through joint inventory, and Alamri [13] created an ideal vendor-buyer model, where a distribution ratio of green production is used as a determinant of joint carbon decisions. Wei et al. [15] studied the optimal environmental product and revenue sharing policy for the cooperative supply chain under cap-and-trade scheme. Liu and Chen [18] studied the optimal ordering and production strategy in a carbon cap-and-trade under a stochastic demand scenario. They have also proven that a sustainable supply chain's performance will also be influenced by carbon regulations, collaboration, coordination, and the power to make decisions shared within the supply chain itself.

2.3 Sustainable inventory models with controllable emissions, deterioration, and green investment

An important portion of the existing literature has focused on sustainable inventory models that incorporate carbon emissions, product deterioration, stock-outs, defective production, and green technology investment in mathematical models. For example, Mashud et al. [21] developed a sustainable inventory model considering manageable carbon emissions, product deterioration, and down payments in their model. Based on these observations, researchers have identified carbon emissions as a controllable factor in inventory systems. Mishra et al. [26] designed a sustainable production-inventory system under the assumption of a controllable rate of carbon emissions in an environment of shortages. The study is also relevant because it uses shortage rate decisions to manage emission levels. Mashud et al. [27] studied a sustainable inventory model with structural defects, deterioration, and controllable emissions. Their study showed that product quality and the rate of deterioration play meaningful roles in determining the costs and sustainability of the inventory. Shah et al. [2] explored adjustable carbon emission inventory models for a deteriorating inventory with a nonlinear deterioration rate, partial backlogging, and demand that is dependent on the selling price. This work further extends the existing literature such as those studied in deterioration, backlogging, responsive to demand, and sustainable carbon emissions. Mishra et al. [25] extended the sustainable supply chain inventory model to controllable rates of deteriorating items and rates of emission in a greenhouse farming environment. The results showed that both the deterioration and emissions rates can be controlled through appropriate operational decisions.

Recent sustainable inventory models also incorporate green investments and green technologies that reduce emissions. Ruidas et al. [19] consider an inventory production model for an eco-friendly product with investment in technologies that reduce emissions and green subsidies. They even pointed out that investing in green technologies can help reduce emissions and build sustainability. Jauhari [17] also presented a sustainable inventory management model for closed-loop supply chains with energy consumption, imperfect production as well as investments in Green technologies. In addition to energy use and green investments, research on reverse logistics, recycling, remanufacturing, and closed-loop supply chains stresses the role of waste recovery and reuse in the management of sustainable supply chains. It is thus recommended that the costs included in inventory models should not only consist of classical costs (ordering costs, holding costs, purchasing costs, shortage costs, backorder costs), but also environmental costs (carbon taxes, carbon costs, investments in sustainability and preservation technologies).

Based on the above literature, it is clear that many researchers have considered the sustainable supply chain, carbon tax schemes, cap-and-trade schemes, green investments, manageable deterioration, emission reductions, reverse logistics, pricing, game theory-based cooperation. However, not a single researcher has studied the construction of a mathematical model at the same time for green degree, investment in conservation technologies, deterioration, partial backlogging, costs of carbon emissions and minimization of total costs. This paper's main contribution is to develop a sustainable inventory model of the product supply chain consisting of permissible carbon emissions. The critical decision variables of the proposed model are the green degree, conservation investment, and the time of starting the shortage of the supply chain.

3. Problem Description

The effects of a carbon tax on the delayed shortage of a product are captured via the market size, consumer preference towards the less polluting types of products, and the price. The effect of time on the quality of a product is modelled according to the investments in preservation technologies. We determine the optimal total cost of the inventory model by selecting the UGD and the investment in preservation technology as decision variables and applying the conventional optimisation method to reach the values of the decision variables that would minimise the total cost. In addition, this study is aligned with objectives of environmental sustainability, resource efficiency, and consumer satisfaction.

4. Notations and Assumptions

a. Notations

All the research parameters are mentioned below for ease of reference.

a Coefficient of the customer’s sensitivity about the green degree.

b Coefficient of customer sensitivity about the selling price

D_0 Fixed demand of the customer

G Green degree

P Selling price

ξ Investment in preservation technology

m(ξ) Reduction in deterioration

θ Original deterioration rate, $0 < \theta < 1$

I_0 Initial inventory level

δ Backlogging parameter

T Cycle length

c Simulation constant

b. Assumptions

The following lists the study's underlying presumptions.

- The demand in inventory models is influenced by consumer demand for environmentally friendly products. So, demand is assumed to be driven by market potential, consumer desire for green products and product pricing, i.e., $D = D_0 + aG - bP$.
- A single product is ordered.
- Shortages are allowed and partially backlogged.
- The model also incorporates the carbon tax policy.
- Product deterioration over time is affected by investment in preservation technology, i.e., ξ, and the decrease in deterioration is given as $m(\xi) = \theta(1 - e^{-\xi})$.

5. Mathematical Framework

Figure 1 depicts the fluctuation in inventory levels over time. Here, $I_0$is the initial inventory level, which decreases because of customer demand and deterioration of goods within the interval $[0, t_1]$. Shortage occurs at time t_1, as demand for the product exceeds the available inventory. These shortages are partially backlogged with parameter δwithin the interval $[t_1, T]$.

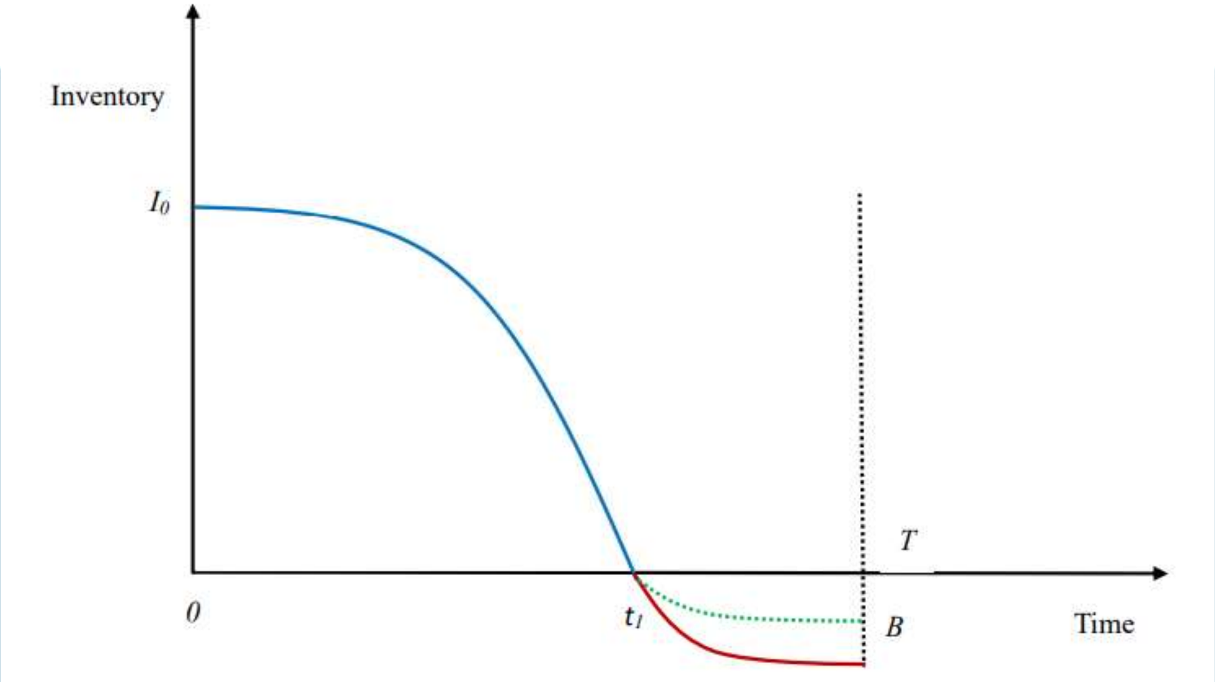


Figure 1. Inventory level.

The mathematical framework is given below.

$$\frac{dI_1(t)}{dt} = -(D_0 + aG - bP) - \big(\theta - m(\xi)\big)I_1(t), 0 \le t \le t_1. \qquad \text{`} \qquad (1)$$

Using boundary condition,

$$I_1(0) = I_0. \qquad (2)$$

$$I_1(t) = -\frac{(D_0+aG-bP)}{\theta(1-e^{-\xi})} + e^{-\theta(1-e^{-\xi})t}\left[\frac{(D_0+aG-bP)}{\theta(1-e^{-\xi})} + I_0\right]. \qquad (3)$$

Using boundary condition,

$$I_1(t_1) = 0. \qquad \textit{(4)}$$

$$I_0 = \frac{(D_0+aG-bP)}{\theta(1-e^{-\xi})}\left(e^{\theta(1-e^{-\xi})t_1}-1\right). \tag{5}$$

$$\frac{dI_2(t)}{dt} = -\frac{(D_0+aG-bP)}{1+\delta(T-t)}, t_1 \le t \le T \tag{6}$$

Using boundary condition,

$$I_2(t_1) = 0. \tag{7}$$

$$I_2(t) = -\frac{(D_0+aG-bP)}{\delta}\log\left[\frac{1+\delta(T-t)}{1+\delta(T-t_1)}\right]. \tag{8}$$

And using boundary condition,

$$I_2(T) = -B. \tag{9}$$

$$B = -\frac{(D_0+aG-bP)}{\delta}\log\left[\frac{1}{1+\delta(T-t_1)}\right]. \tag{10}$$

The quantity ordered is represented as Q.

$$Q = I_0 + B. \tag{11}$$

Using equation (5) and (10),

$$Q = (D_0 + aG - bP)\left[\frac{e^{\theta(1-e^{-\xi})t_1}-1}{\theta(1-e^{-\xi})} - \frac{1}{\delta}\log\left(\frac{1}{1+\delta(T-t_1)}\right)\right]. \tag{12}$$

6. Cost Calculation

i. Ordering Cost

Ordering cost, also known as procurement or setup cost, refers to the expenses incurred in placing and receiving orders for new inventory, including administrative costs like paperwork, communication, and labour involved in preparing and processing purchase orders. The ordering cost for the model is given as:

$$OC = \frac{(O+O')}{T}. \tag{13}$$

where O is the ordering cost per order, and O' is the carbon emission cost during ordering.

ii. Holding Cost

Holding cost, or carrying cost, is the cost a business incurs for maintaining or storing inventory over a specific period, including rent, expenses for physical space, warehouse space, utilities, and necessary equipment, all essential for maintaining stock. The holding cost of the model is given as:

$$HC = \frac{(h+h')}{T}\int_0^{t_1} I_1(t)\,dt. \tag{14}$$

where h is the unit holding cost, and h' is the unit carbon emission cost while maintaining inventory.

$$C = \frac{(h+h')}{T(1-e^{-\xi})}\left[\left\{I_0+\frac{(D_0+aG-bP)}{\theta(1-e^{-\xi})}\right\}\left(1-e^{-\theta(1-e^{-\xi})t_1}\right)-(D_0+aG-bP)t_1\right]. \tag{15}$$

iii. Purchasing Cost

The purchase cost is the company’s money on its products or goods. The purchasing cost for the model is given as:

$$PC = \frac{p_c}{T}Q. \tag{16}$$

where p_c is the unit purchasing cost.

$$PC = \frac{p_c}{T}(D_0+aG-bP)\left[\frac{e^{\theta(1-e^{-\xi})t_1-1}}{\theta(1-e^{-\xi})}-\frac{1}{\delta}\log\left(\frac{1}{1+\delta(T-t_1)}\right)\right]. \tag{17}$$

iv. Shortage Cost

Shortage cost refers to the negative consequences and expenses a business incurs due to insufficient inventory to meet customer demand, resulting from running out of goods. The shortage cost for the model is given as:

$$SC = -\frac{s_c}{T}\int_{t_1}^{T} I_2\,(t)\,dt. \tag{18}$$

where $s_c$is unit shortage cost.

$$SC = \frac{s_c(D_0+aG-bP)}{T\delta}\left[T - t_1 - \frac{1}{\delta}\log\{1+\delta(T-t_1)\}\right]. \tag{19}$$

v. Lost Sale Cost

The financial loss a business experiences when it cannot meet consumer demand because of insufficient inventory, leading to potential customers departing without purchasing the products they want, is known as lost sales cost. The lost sales cost is given as:

$$LS = \frac{l_s}{T}\int_{t_1}^{T}\left[1 - \frac{1}{1+\delta(T-t)}\right](D_0 + aG - bP)\,dt. \tag{20}$$

where $l_s$is unit lost sales cost.

$$LS = \frac{l_s(D_0+aG-bP)}{T}\left[T - t_1 - \frac{1}{\delta}\log\{1+\delta(T-t_1)\}\right]. \tag{21}$$

vi. Preservation Investment

The expense incurred by a company to preserve and safeguard the quality of its inventory over time, including the use of technologies or treatments and resources to keep goods in excellent condition while being kept or held for sale, is known as the preservation investment in an inventory model. The preservation investment is given as:

$$PT = \frac{(\xi + \xi')}{T}. \tag{22}$$

where ξ is a preservation technology investment and ξ' is carbon emission cost during preservation.

The total cost is all the costs related to purchasing, storing, and overseeing inventory within an organization. It includes a range of expenses incurred throughout inventory management. Companies usually want to keep inventory costs as low as possible while having enough products to satisfy customer demand. The total cost is given as:

$$TC = OC + HC + PC + SC + LS + PT. \tag{23}$$

$$\begin{aligned} TC(\xi, G, t_1) = {} & \frac{(O+O')}{T} + \frac{(h+h')}{T(1-e^{-\xi})}\left[\left\{I_0 + \frac{(D_0+aG-bP)}{\theta(1-e^{-\xi})}\right\}\left(1-e^{-\theta(1-e^{-\xi})t_1}\right) - (D_0+aG-bP)t_1\right] \\ & + \frac{p_c}{T}(D_0+aG-bP)\left[\frac{e^{\theta(1-e^{-\xi})t_1}-1}{\theta(1-e^{-\xi})} - \frac{1}{\delta}\log\left(\frac{1}{1+\delta(T-t_1)}\right)\right] \\ & + \frac{s_c(D_0+aG-bP)}{T\delta}\left[T - t_1 - \frac{1}{\delta}\log\{1+\delta(T-t_1)\}\right] \\ & + \frac{l_s(D_0+aG-bP)}{T}\left[T - t_1 - \frac{1}{\delta}\log\{1+\delta(T-t_1)\}\right] + \frac{(\xi+\xi')}{T}. \end{aligned} \tag{24}$$

7. Solution Methodology

The approach for resolving the optimization issues with the inventory model is presented in this section. The objective is to minimize the total cost function per unit of time. According to the suggested model, the total cost equations contain three independent variables, ξ, G, and t_1. The following steps are taken to obtain all independent parameters' values and optimize the total cost equation.

Step 1. Set the first-order partial derivatives of the total cost function TC with respect to ξ, G, and t_1 equal to zero and solve to determine their value.

$$\frac{\partial TC}{\partial \xi} = 0, \frac{\partial TC}{\partial G} = 0, \frac{\partial TC}{\partial t_1} = 0. \qquad (25)$$

Step 2. With respect to each independent variable, calculate the second-order partial derivatives.

Step 3. Create a Hessian matrix as shown below.

$$H = \begin{bmatrix} \frac{\partial^2 TC}{\partial \xi^2} & \frac{\partial^2 TC}{\partial \xi\, \partial G} & \frac{\partial^2 TC}{\partial \xi\, \partial t_1} \\ \frac{\partial^2 TC}{\partial G\, \partial \xi} & \frac{\partial^2 TC}{\partial G^2} & \frac{\partial^2 TC}{\partial G\, \partial t_1} \\ \frac{\partial^2 TC}{\partial t_1\, \partial \xi} & \frac{\partial^2 TC}{\partial t_1\, \partial G} & \frac{\partial^2 TC}{\partial t_1^2} \end{bmatrix}. \qquad (26)$$

Step 4. The first principal minor, second principal minor, and third principal minors are denoted by H_1, H_2, and H_3, respectively. The principal minors $\mid H_1 \mid > 0$, $\mid H_2 \mid > 0$, and $\mid H_3 \mid > 0$must show a positive sign to minimize the inventory system's overall cost function. If the condition is satisfied, the total cost function has a global minimum at the optimum value of the decision variables.

8. Numerical Analysis

The optimal total cost has been developed in this section with a numerical example to validate the model. The values of the parameters are sourced from (Poswal et al., 2022b). Ordering cost $O = \$400/order$, carbon emission cost per order $O' = \$5/order$, per unit holding cost $h = \$4/unit$, per unit carbon emission cost while having inventory $h' = \$0.3/unit$, simulation constant $c = 0.09$, fixed demand of the customer $D_0 = 75$units, coefficient of the customer's sensitivity about the green degree $a = 5$, coefficient of customer sensitivity about the selling price $b = 4$, coefficient of customer sensitivity about the selling price $P = \$20$, original deterioration rate $\theta = 0.5$, per unit purchasing cost $p_c = \$14/unit$, backlogging parameter $\delta = 0.4$, per unit shortage cost $s_c = \$6/unit$, per unit lost sales cost $l_s = \$5/unit$, carbon emission cost during preservation $\xi' = \$0.9$, cycle length $T = 18$weeks.

Table 1 illustrates the optimal values of the decision variables by applying the methodology mentioned above to the data. The higher green degree implies a product's lower carbon footprint, as it is made from environmentally friendly materials and designed to minimize environmental impact throughout its lifecycle.

Table 1. Decision variable optimal values

G	ξ ($)	t_1 (weeks)	TC ($)
0.8404	772.3800	8.0000	25851.5000

9. Sensitivity Analysis

Sensitivity analysis plays a crucial role in assessing the resilience of a supply chain by pinpointing essential factors that affect system dynamics and offering a comprehensive insight into the input variables and their resulting effects. Table 2 demonstrates the responsiveness of the essential parameters.

Table 2. Sensitivity analysis for crucial parameters

Parameter	% Change	G	ξ ($)	t_1 (weeks)	TC ($)
Ordering cost O'	-20	0.8404	974.3700	8.0000	21408.1021
	-10	0.8404	842.8300	8.0000	23908.1120
	0	0.8404	772.3800	8.0000	25851.5000

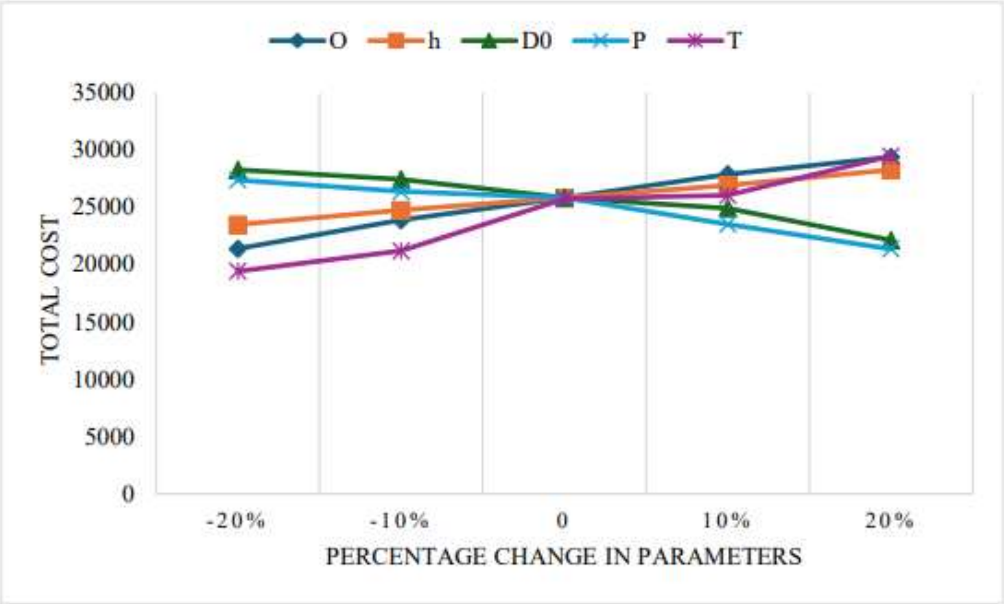
	10	0.8404	574.9500	8.0000	27908.1135
	20	0.8404	476.8800	8.0000	29412.6513
Unit holding cost h'	-20	0.8404	824.2045	7.0842	23501.3546
	-10	0.8404	797.9946	7.8822	24790.4213
	0	0.8404	772.3800	8.0000	25851.5000
	10	0.8404	747.2512	8.5789	26968.0213
	20	0.8404	722.5022	8.6619	28298.3213
Fixed demand D_0	-20	1.9946	772.3800	8.0000	28308.1516
	-10	1.5054	772.3800	8.0000	27489.2213
	0	0.8404	772.3800	8.0000	25851.5000
	10	0.4946	772.3800	8.0000	24956.1213
	20	0.0054	772.3800	8.0000	22154.8564
Selling price P	-20	0.6915	772.3800	7.7169	27408.1564

	-10	0.7265	772.3800	7.8608	26395.4545
	0	0.8404	772.3800	8.0000	25851.5000
	10	0.8603	855.5351	8.8608	23548.9446
	20	0.8736	855.5351	8.8608	21395.4246
Deterioration rate θ	-20	0.8407	591.6356	8.0000	22452.4546
	-10	0.8405	627.1354	8.0000	24720.7545
	0	0.8404	772.3800	8.0000	25851.5000
	10	0.8403	878.7654	8.0000	26977.4546
	20	0.8401	998.1254	8.0000	28147.1859
Unit purchasing cost p^c	-20	0.8404	770.3716	8.8001	24431.3546
	-10	0.8404	771.9921	8.7808	25183.6845
	0	0.8404	772.3800	8.0000	25851.5000
	10	0.8404	776.9863	7.9401	26276.7746

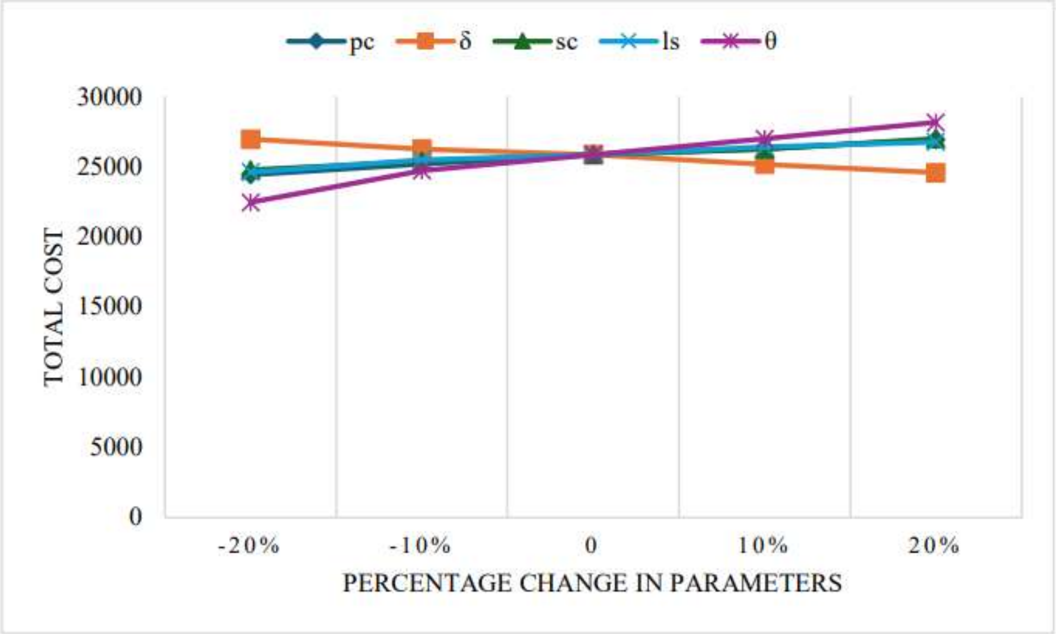
	20	0.8404	778.3632	7.0186	26917.6454
Backlogging parameter δ	-20	0.5430	785.6926	7.0258	26954.5546
	-10	0.8533	773.1462	7.9353	26258.7546
	0	0.8404	772.3800	8.0000	25851.5000
	10	0.8659	741.7456	8.7982	25164.4455
	20	0.8701	728.6984	8.7449	24575.4451
Unit shortage cost s^c	-20	0.8404	0.8563	7.7162	24764.7646
	-10	0.8404	0.8559	7.7880	25382.5213
	0	0.8404	0.8404	8.0000	25851.5000
	10	0.8404	0.8400	8.9348	26251.5323
	20	0.8404	0.8397	9.0099	26994.4416
Unit lost sales cost l_s	-20	0.8404	739.5451	7.7213	24616.4559
	-10	0.8404	750.0816	7.8364	25495.1446

	0	0.8404	772.3800	8.0000	25851.5000
	10	0.8404	780.8254	8.8853	26395.1546
	20	0.8404	795.9762	8.9100	26754.4628
Cycle length T	-20	0.8584	671.8125	6.5778	19446.0216
	-10	0.8556	782.5512	7.7326	21235.8213
	0	0.8404	772.3800	8.0000	25851.5000
	10	0.8401	908.0513	8.9716	26074.3231
	20	0.8321	949.9465	9.0702	29521.6462

The information provided in Table 2 has been condensed below and illustrated in Figure 2.


O
h
D0
P
T
35000
30000
25000
20000
15000
10000
5000
0
TOTAL COST
-20%
-10%
0
10%
20%
PERCENTAGE CHANGE IN PARAMETERS

(a)


pc
δ
sc
ls
θ
30000
25000
20000
15000
10000
5000
0
TOTAL COST
-20%
-10%
0
10%
20%
PERCENTAGE CHANGE IN PARAMETERS

(b)

Figure 2. Sensitivity analysis of different parameters.

i. Total cost increases with an increase in the ordering cost. Ordering costs and total costs, of course, are also inter-related and change depending upon market conditions, technology and corporate policy. Thus, when ordering costs increase, so do holding costs, storage costs, and obsolescence costs.

ii. The total cost increases slowly as the holding cost rises. Particularly in a manufacturing environment, care should be taken by managers to determine appropriate levels of safety stock to meet demand without incurring excessive holding costs for excess inventory.

iii. The overall cost decreases as the demand parameter increases. It allows retailers to identify opportunities for improving operational efficiency, lowering costs, and better planning and managing inventory and suppliers.

iv. With increase of selling price, the total costs are substantially reduced. When retailers can raise margins through higher prices, they have more money to reinvest in their business, leading to higher profits, which in turn allows them to invest in greater customer service, quality and long-term customer satisfaction.

v. The cost of producing output rises considerably as the length of the cycle grows longer. The longer the inventory cycle the bigger the carrying cost, the larger the amount of capital tied up, and the more other costs or opportunities are lost, e.g. warehousing costs or coverage costs. Long inventory cycle times tie up working capital and resources.

vi. Acquisition costs have little effect on total spending in this analysis, though they are increasing. Negotiations with suppliers are also part of responding to market conditions, contributing to cost, and reaching a satisfactory agreement, particularly with regards to changing suppliers or lowering costs.

vii. As the backlogging parameters increase, the total cost of operation decreases. In this manner, the backlogging parameter potentially allows companies to be more responsive to changes in demand without creating surplus, if inventory holding costs are incurred due to overproduction.

viii. The overall cost increases with an increase in shortage cost. To avoid stockouts and costs, companies need to invest in effective demand forecasting and inventory management, so that they can fulfil customer demand at every stage of the supply chain.

ix. An increase in lost sales expense slightly raises expenses for this period. Opportunity costs within the company are rising, as well as customer satisfaction, brand equity etc. Maintaining good relations with suppliers, and correctly anticipating demand and controlling inventories are vital to offset the costs of stock-outs and other consequences of poor supply chain management.

x. The overall cost associated with the operation of the model depends heavily on the rate of decline. This includes, but is not limited to, keeping an eye on market trends, forecasting demand and determining when to end the life of a product.

10. Discussion

According to the results of the calculations, the use of the preservation technology and increase in green rating lead to a decrease of the economic costs of the whole supply chain and of the carbon emissions. The increase of green rating means that the product is made from green materials and leads to a decrease of ecological footprint for the whole lifetime of the supply chain. This confirms the need for a sustainable supply chain network design for industrial manufacturing that focuses on reducing carbon emissions intentionally [3]. The model's inclusion of market potential and environmental sustainability shows that the selection of supply chain strategies incorporating sustainability aspects can lead to increased efficiency, customer loyalty, and a stronger competitive position within the company. The main contribution of this model is its analytical treatment of carbon emissions and product deterioration as operational variables, rather than uncontrollable waste. In addition, our understanding indicates that the cost of preservation technologies is an important operational cost to incur, as it has the potential to reduce deterioration considerably. Therefore, this is related to the current studies to determine the most effective regulations on carbon emissions for sustainable inventory models with gradual deterioration, price-sensitive demand, and partial backlogging [2]. The effective regulation of these factors leads to a reduction in the ecological and economic effects of the unsold expired items and the related emissions.

According to the extreme sensitivity analysis of the system, it can be concluded that cycle length and product decay rate are the most sensitive variables influencing total costs of the system. Total costs of the system will increase at a low rate with increasing holding costs of the unit. These fluctuations mean that inventory managers are constantly striving to optimise safety stock levels to balance the cost of holding stock which may end up spoiling, against the cost of stock outs. This leads to the need to model such fluctuations and to apply strong optimisation models to dual-objective forward and reverse logistics networks where uncertainty, carbon emissions and storage efficiency must be managed collaboratively [1]. The importance of the relationship between the selling price, the demand and the total cost of the system is stressed from a financial point of view, as the selling price can contribute to the reduction of total costs. Increased income makes it possible to invest in a sustainable supply chain network structure, more ecological products, and long-term customer service improvements. The additional gross profit per product sold enables the company to absorb the short-term costs of sustainable investment, and thus the transition to more sustainable business practices can be undertaken without impacting overall profitability.

The implementation of the carbon tax component into the inventory model has proven that regulatory compliance can be achieved at a lower cost through an optimal investment in energy-efficient conservation measures compared to facing a high carbon tax penalty or incurring the costs associated with energy shortages and lost sales opportunities. The theoretical studies cited earlier in this section have shown that inventory models with green items and investment in technology for emission and pollutant reduction are long-term viable solutions for supply chains [19]. Thus, the proposed model offers several advantages to companies by allowing them to reduce their GHG emissions while simultaneously managing their procurement, inventory, and ordering costs. The mathematical model presents an extraordinarily realistic and coherent structure of modern sustainable supply chain management, which takes into consideration the partial backlogging, the optimising of the green degrees and all the carbon emission costs. The optimization results, confirmed by the Hessian matrix method, prove that the minimum global cost can be achieved by knowing when the shortage can occur and which preservation technologies should be used. Our study shows environmental and economic sustainability could mutually coexist and even be mutually supportive. The eco-friendly inventory process proposed in this study gives companies more efficient use of resources, creates greater customer satisfaction and makes it easier to meet sustainability requirements for the environment.

11. Conclusion

The work contributes to research on customary industry, eco-friendly behaviour, and regulation, and proposes a decision-making framework for managing inventory when accounting for product degradation and temporary shortages in inventory. In quantitative terms, we achieved high eco-friendliness since its carbon footprint is low. Sensitivity analysis indicates that the safety stock should be adjusted with care by the manager to ensure product availability and at the same time save costs, as total cost increases slowly as the holding costs increase. Cycle length is extremely sensitive. Higher prices reduce total cost per period, allowing the retailer to cycle fast and thus amass more profits per period for reinvestment. This would encourage companies to invest in long-term customer satisfaction, better service and improved quality. Future studies could investigate the impact of social media on consumers' intentions and attitudes to purchase green products.

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